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Applying Linguistic Cognitive Map Method to Deal with Multiple Criteria Decision-making Problems

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In general, decision-makers must consider many influence factors when dealing with decision-making problems, and these factors will interact with each other. The fuzzy cognitive map (FCM) is an analysis tool that can illustrate the causal relationships among influence factors by a network structure. In addition, the opinions of experts are subjective and vague in the decision-making process. It is suitable for experts to use linguistic variables to express their opinions. Therefore, this paper presented a linguistic cognitive map decision method (LCMDM) by combining linguistic variables with a fuzzy cognitive map to deal with multiple criteria decision-making problems. Finally, a numerical example was implemented to illustrate the computational process of the proposed method. The conclusions and future research directions were discussed at the end of this paper.

Keywords: linguistic variables, fuzzy cognitive map, multiple criteria decision making (MCDM)

JEL classification: C00, C02, C18

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1 Introduction

In the real environment, experts and decision-makers must cope with vague and imprecise information, which often involves uncertainty in the decision-making process (Martinez and Herrera, 2012). It is not always adequate to represent uncertain information using crisp numerical values. Under this situation, it is suitable for experts and decision-makers to express their opinions by using linguistic values in decision-making problems (Martinez *et al*., 2005; Martinez *et al*., 2009).

In the decision-making process, there are many influence factors that must be considered when dealing with decision-making problems, and these factors will interact with each other. Due to the assumption of independence among factors, it is difficult to deal with interactive relationships using traditional MCDM methods. Therefore, the fuzzy cognitive map (FCM) is a suitable tool for handling this problem. Fuzzy cognitive maps are symbolic representations of the description and modeling of a complex system (Kosko, 1996). Fuzzy cognitive maps have been applied in numerous areas, such as exploratory studies of solar energy (Jetter and Schweinfort, 2011), media adaptation in tourism web sites (Kardaras *et al*., 2013), business performance measurement (Glykas, 2013), system control (Stylios and Groumpos, 2000), ecosystem conservation (Özesmi and Özesmi, 2003), and agricultural applications (Papageorgiou *et al*., 2011). Fuzzy cognitive maps also have been used for decision analysis and operation research (Glykas, 2010; Salmeron, Jose and Lopez, 2010; Georgopoulos and Stylios, 2015). Therefore, the aim of this paper was to present a novel and comprehensive method called the linguistic cognitive map decision method (LCMDM) by combining linguistic variables with a fuzzy cognitive map to deal with MCDM problems. LCMDM contains three sub processes to deal with decision-making problems in a fuzzy environment: the target system building process, the aggregation process, and the cognitive map decision process. The advantage of the proposed method is that it is easy to compute the status values of all criteria during decision-making at different periods. Under this situation, it could help the decision-maker reach a reasonable decision at different time points.

In this paper, linguistic variables were used for experts to express the degree of relationship between two factors (or criteria) and the initial status of each factor (or criterion). The notations and the computation formulas of the linguistic variables were presented for the experts to express their opinions, and an aggregation method was developed in order to aggregate the opinions of multiple decision-makers or experts and acquire the group opinion. Then, the relation matrix of the fuzzy cognitive map was constructed to describe the relationships of all factors. A numerical example was presented to illustrate the computational process of proposed method. Finally, the conclusions and future research were discussed at the end of this paper.

2 Linguistic Variable

In general, experts cannot express their opinions using crisp values; however, they can use linguistic variables to express their opinions easily. For example, the importance of evaluation criterion can be expressed as "very important" or "of little importance". Therefore, linguistic variables can be used to express the opinions of experts for the status values of all criteria and the degrees of relationship among all criteria.

Definition 1: Let $S^g = \{s^g_{-\frac{g-1}{2}}, s^g_{-\frac{g-1}{2}+1}, \ldots, s^g_{-1}, s^g_0, s^g_1, \ldots, s^g_{\frac{g-1}{2}-1}, s^g_{\frac{g-1}{2}}\}$ be a finite and totally ordered linguistic term set in intervals of [-1, 1]. The membership function of each linguistic variable can be represented as a triangular fuzzy number. The linguistic variable can be expressed as s^g_t, which is the central value of the t-th linguistic term respectively in S^g (Herrera and Martinez, 2001; Herrera, 2012).

Definition 2: The linguistic transfer function Δ^{-1} can translate the linguistic variable into crisp value β ($\beta \in [-1, 1]$), which can be shown as (Tai and Chen, 2009; Chen *et al*., 2013):

$$\Delta^{-1}\left(s^g_t\right) = \beta = \frac{t*2}{g-1}, \tag{1}$$

where, g is the scale of the linguistic term and

t= $\frac{-(g-1)}{2}$, $\frac{-(g-1)}{2}+1$,...,-1,0,1,…, $\frac{g-1}{2}-1$, $\frac{g-1}{2}$.

Definition 3: The symbolic translation function Δ is used to translate crisp value β (β∈ [-1, 1]) into the linguistic variable, which can be shown as (Tai and Chen, 2009; Chen *et al*., 2013):

$$\Delta(\beta) = s^{g}_{\beta * 0.5 * (g-1)}. \quad (2)$$

Definition 4: Assume that L={L_i|i=0,1,...,t-1,t} is a finite, discrete, and complete order set in intervals of [0, 1] (Xu, 2009; Chen *et al*., 2013). Decision makers can use linguistic variables to express their opinions for the status of each criterion in the cognitive map network. For example, the seven scale linguistic variables are shown in Table 1 and their membership functions can be shown as Fig 1.

Table 1. Different Linguistic Variables for the Status Value

	Status value
L_0	Pretty Poor (PP)
L_1	Very Poor (VP)
L_2	Poor (P)
L_3	Medium (M)
L_4	Good (G)
L_5	Very Good V(G)
L_6	Pretty Good (PG)

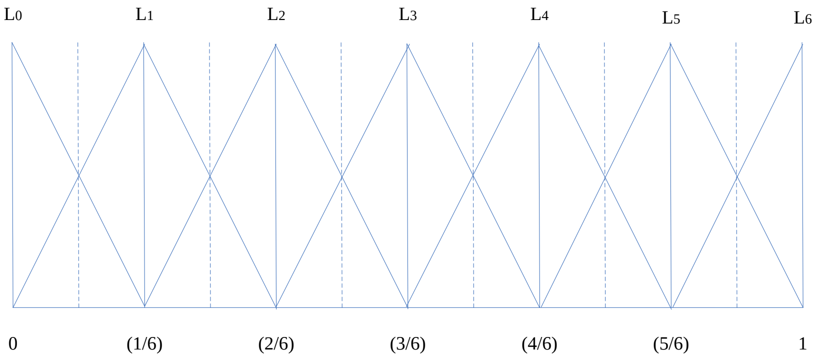

Fig 1. Membership Functions of Linguistic Variable for the Status Value

Definition 5: Assume that L_i is an evaluation linguistic variable. The linguistic variable normalization function for performance or weight Δ_{pw}:L_i->[0,1] can transfer the evaluation linguistic variable into crisp value β ($\beta \in [0,1]$). The linguistic variable normalization function for performance or weight Δ_{pw} can be shown as (Xu, 2004):

$$\Delta_{pw}(L_i) = (i)/(t), \tag{3}$$

where, $L_i \in \bar{L} = \{L_g | g \in [0, t]\}$.

Definition 6: The mean function Θ is used to aggregate the linguistic opinion of each expert as (Chen *et al.*, 2013; Tai and Chen, 2009):

$$\tilde{x} = \Theta\left(\tilde{x}^1, \tilde{x}^2, ..., \tilde{x}^V\right) = \Delta\left(\sum_{k=1}^{V} \Delta^{-1}\left(\tilde{x}^k\right) / V\right), \tag{4}$$

where, $\tilde{x}^k$ represents the k-th expert opinion, which can be indicated as a linguistic variable, and $\tilde{x}$ represents the group opinion of experts and V represents the number of experts.

3 Linguistic Cognitive Map Decision Process

The linguistic cognitive map decision process includes three sub processes, such as the build process of the target system, the expert opinion aggregation process, and the cognitive map decision process.

(1) The target system build process

The build process of the target system includes nodes (system variables) and arcs (degree of relation). X_i is the symbol of node *i*, and the arc e_{ij} is the symbol of the relationship between nodes X_i and X_j.

(2) The expert opinion aggregation process

The initial status of node X_i can be represented as the linguistic variable ($\tilde{x}_i$). The initial status of node X_i of expert k can be represented as the linguistic variable ($\tilde{x}_i^k$). The relation degree (e_{ij}) between nodes X_i and X_j can be represented as linguistic variable ($\tilde{e}_{ij}$). The relation degree between nodes X_i and X_j of expert k can be represented as the linguistic variable ($\tilde{e}_{ij}^k$).

In the expert opinion aggregation process, $\tilde{x}_i^k$ and $\tilde{e}_{ij}^k$ will be aggregated as $\tilde{x}_i$ and $\tilde{e}_{ij}$ by the expert opinion aggregation mechanism. The group opinion of $\tilde{x}_i$ and $\tilde{e}_{ij}$ can be calculated as:

$$\tilde{x}_i = \Theta\left(\tilde{x}_i^1, \tilde{x}_i^2, ..., \tilde{x}_i^v\right) = \Delta\left(\sum_{k=1}^{v} \Delta^{-1}\left(\tilde{x}_i^k\right) / v\right), \quad (5)$$

$$\tilde{e}_{ij} = \Theta\left(\tilde{e}_{ij}^1, \tilde{e}_{ij}^2, ..., \tilde{e}_{ij}^v\right) = \Delta\left(\sum_{k=1}^{v} \Delta^{-1}\left(\tilde{e}_{ij}^k\right) / v\right), \quad (6)$$

where, $\tilde{x}_i^k$ represents the opinion of the initial status of node X_i expressed by expert k, $\tilde{e}_{ij}^k$ represents the opinion of the relation degree between nodes X_i and X_j expressed by expert k, and v represents the number of experts.

(3) The cognitive map decision process

In the cognitive map decision process, the computational steps can be described as follows (Lopes *et al*., 2013):

(i) Transfer the initial status of each node:

$$x_i = \Delta^{-1}\left(\tilde{x}_i\right), \quad (7)$$

where, x_i represents the normalized initial value of node X_i, $0 \le x_i \le 1$, then transfer the relation degree between each node:

$$e_{ij} = \Delta^{-1}\left(\tilde{e}_{ij}\right), \quad (8)$$

where, e_{ij} represents the relation degree between node X_i and node X_j, which is expressed by crisp value $0 \le e_{ij} \le 1$.

(ii) Choose the threshold function for avoiding the value of the node exceeding the interval [0, 1]. The linear threshold function was used in this study.

(iii) System calculation. $E = \lfloor e_{ij} \rfloor$ is the relation matrix, which records the relation degree between node X_i and node X_j, and $V = \lceil x_i \rceil$ is the status matrix, which represents the status of each node X_i. Let V^t be the new status of each node at the t-th period. The linear threshold function can apply to compute the new status matrix, where c is the number of variables in the system.

$$V^t = \frac{E * V^{t-1} + c}{2 * c}. \tag{9}$$

4 Numerical Example

In this example, an enterprise wanted to outsource its enterprise resource planning information system to a cloud service provider. Five criteria were selected to evaluate the outsourcing possibility of the cloud service provider, such as system security (C_1), flexibility (C_2), system quality (C_3), cost (C_4), skill ability (C_5), and possibility of outsourcing (C_6). The enterprise hired three experts to judge the status of each criterion and the relationship among the criteria to evaluate the outsourcing possibility of the cloud service provider. The interactions of each pair of criteria were considered in the evaluation process, as shown in Fig. 2. For example, if the status value of system security (C_1) increased then the status value of flexibility (C_2) would decrease. Therefore, the relationship was negative from system security (C_1) to flexibility (C_2).

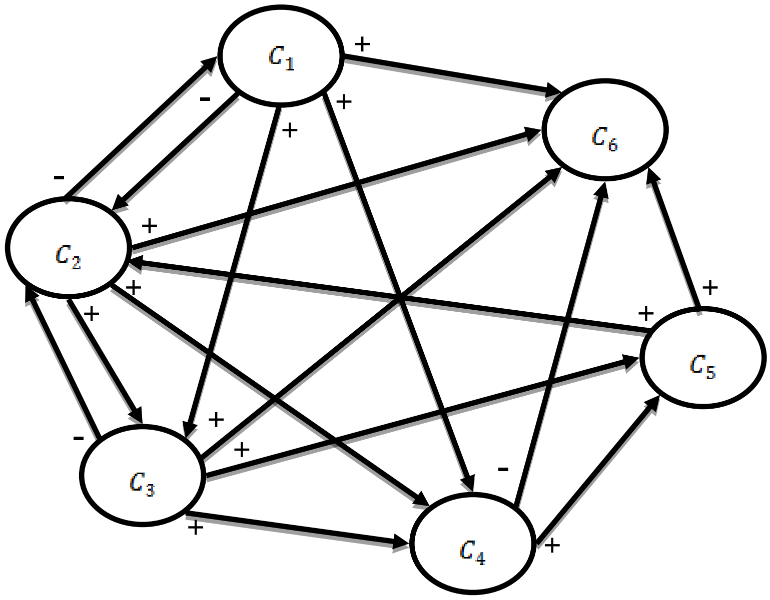

Fig. 2. The Interactions among the Criteria

The computational process of the linguistic cognitive map decision process could be illustrated as follows.

Step 1: The experts used linguistic variables (Table 2) to express their opinions. Expert 1 (E_1) used five scale linguistic variables, expert 2 (E_2) used seven scale linguistic variables, and expert 3 (E_3) used nine scale linguistic variables to express their opinions. The relation degree between two criteria of each expert are shown as Table 3, Table 4 and Table 5.

Step 2: The experts used linguistic variables (Table 2) to express the initial status of each criterion, as shown in Table 6.

Step 3: The aggregated values of the relation degree between two criteria of the three experts were as shown in Table 7, and the aggregated initial status values of three experts were as shown in Table 8.

Step 4: The status of each criterion at each period could be calculated in accordance with equation (9) and the results could be shown as Table 9 and Fig. 3. The final status of each criterion of the cloud service provider could be shown as Table 10. According to Table 10, the possibility of outsourcing to the provider was 0.6304 and the order of the status importance of all criteria was cost (C_4 , 0.6670) >

skill ability (C_5, 0.5961) > system quality (C_3, 0.5888) > system security (C_1, 0.5479) > system flexibility (C_2, 0.5268). According to the final status of each criterion, most of the criteria would increase the importance of evaluating the cloud service providers. However, the increment degree of importance was different for each criterion. Among them, the increment degree of importance for cost (C_4) was more relatively evident than the other criteria.

The advantage of the proposed method is that it can easily express the opinions of experts by using different linguistic variables and compute the status values of all criteria in decision-making at different periods. Under this situation, it could help decision-makers reach a reasonable decision at different time points.

Table 2. Different Types of Linguistic Variables

		Linguistic variable
Five Scale of Linguistic Variables	Relation Value	High Negative Relation(HNR) $\left(s_{-2}^{5}\right)$、Low Negative Relation(LNR) $\left(s_{-1}^{5}\right)$、Ordinary(O) $\left(s_{0}^{5}\right)$、Low Positive Relation(LPR) $\left(s_{1}^{5}\right)$、High Positive Relation(HPR) $\left(s_{2}^{5}\right)$
	Status Value	Very Poor (VP)(L_0^5)、Poor(P) (L_1^5)、Medium(M) (L_2^5)、Good(G) (L_3^5)、Very Good(VG) (L_4^5)
Seven Scale of Linguistic Variables	Relation Value	High Negative Relation(HNR) $\left(s_{-3}^{7}\right)$、Negative Relation(NR) $\left(s_{-2}^{7}\right)$、Low Negative Relation(LNR) $\left(s_{-1}^{7}\right)$、Ordinary(O) $\left(s_{0}^{7}\right)$、Low Positive Relation(LPR) $\left(s_{1}^{7}\right)$、Positive Relation(PR) $\left(s_{2}^{7}\right)$、High Positive Relation(HPR) $\left(s_{3}^{7}\right)$
	Status Value	Pretty Poor(PP) (L_0^7)、Very Poor(VP) (L_1^7)、Poor(P) (L_2^7)、Medium(M) (L_3^7)、Good(G) (L_4^7)、Very Good(VG) (L_5^7)、Pretty Good(PG) (L_6^7)
Nine Scale of Linguistic Variables	Relation Value	Extremely High Negative Relation(EHNR) $\left(s_{-4}^{9}\right)$、High Negative Relation(HNR) $\left(s_{-3}^{9}\right)$、Negative Relation(NR) $\left(s_{-2}^{9}\right)$、Low Negative Relation(LNR) $\left(s_{-1}^{9}\right)$、Ordinary(O) $\left(s_{0}^{9}\right)$、Low Positive Relation(LPR) $\left(s_{1}^{9}\right)$、Positive Relation(PR) $\left(s_{2}^{9}\right)$、High Positive Relation(HPR) $\left(s_{3}^{9}\right)$、Extremely High Positive Relation(EHPR) $\left(s_{4}^{9}\right)$
	Status Value	Extremely Poor(EP) (L_0^9)、Pretty Poor(PP) (L_1^9)、Very Poor(VP) (L_2^9)、Poor(P) (L_3^9)、Medium(M) (L_4^9)、Good(G) (L_5^9)、Very Good(VG) (L_6^9)、Pretty Good (PG) (L_7^9)、Extremely Good (EG) (L_8^9)

Table 3. The Relation Degree Matrix of Expert 1

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	O	LNR	LPR	HPR	O	HPR
C_2	LNR	O	LPR	HPR	O	HPR
C_3	LPR	LNR	O	LPR	LPR	LPR
C_4	O	O	O	O	LPR	HNR
C_5	O	LPR	O	O	O	LPR
C_6	O	O	O	O	O	O

Table 4. The Relation Degree Matrix of Expert 2

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	O	NR	LPR	HPR	O	HPR
C_2	LNR	O	LPR	HPR	O	HPR
C_3	LPR	LNR	O	LPR	LPR	LPR
C_4	O	O	O	O	LPR	HNR
C_5	O	PR	O	O	O	LPR
C_6	O	O	O	O	O	O

Table 5. The Relation Degree Matrix of Expert 3

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	O	NR	PR	EHPR	O	EHPR
C_2	NR	O	PR	EHPR	O	EHPR
C_3	PR	LNR	O	PR	PR	PR
C_4	O	O	O	O	PR	EHNR
C_5	O	PR	O	O	O	PR
C_6	O	O	O	O	O	O

Table 6. The Initial Linguistic Status of Each Criterion

	C_1	C_2	C_3	C_4	C_5	C_6
E_1	P	M	G	VP	M	VP
E_2	P	M	G	VP	M	PP
E_3	P	M	G	PP	M	EP

Table 7. The Aggregated Relation Degree Matrix by Representing as the Crisp Values

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	0	-0.5567	0.4433	1	0	1
C_2	-0.4433	0	0.4433	1	0	1
C_3	0.4433	-0.36	0	0.4433	0.4433	0.4433
C_4	0	0	0	0	0.4433	-1
C_5	0	0.5567	0	0	0	0.4433
C_6	0	0	0	0	0	0

Table 8. The Aggregated Initial Status of Each Criterion by Crisp Values

C_1	C_2	C_3	C_4	C_5	C_6
0.318	0.5	0.722	0.097	0.5	0

Table 9. The Status of Each Criterion at Each Period

	C_1	C_2	C_3	C_4	C_5	C_6
Initial status	0.318	0.5	0.722	0.097	0.5	0
round 1	0.5347	0.5284	0.5904	0.6029	0.5719	0.6052
round 2	0.5468	0.5281	0.5885	0.6607	0.5917	0.6317
round 3	0.5478	0.5284	0.5888	0.6664	0.5955	0.6308
round 4	0.5479	0.5286	0.5888	0.6670	0.5960	0.6305
round 5	0.5479	0.5286	0.5888	0.6670	0.5961	0.6304
round 6	0.5479	0.5286	0.5888	0.6670	0.5961	0.6304

Table 10. The Final Status of Each Criterion

C_1	C_2	C_3	C_4	C_5	C_6
0.5479	0.5268	0.5888	0.6670	0.5961	0.6304

5 Conclusion

In general, numerous influencing factors may interact with each other when dealing with MCDM problems. The fuzzy cognitive map method is suitable to handle these problems. In addition, the opinions of experts may be subjective, vague, and fuzzy. Linguistic variables are suitable for expressing expert evaluations. This paper presented the linguistic cognitive map decision method (LCMDM) to cope with MCDM problems in a fuzzy environment. The advantage of the proposed method is that it can easily express the opinions of experts by using different linguistic variables and compute the status values of all criteria in decision-making at different periods. Under this situation, it could help decision-makers reach a reasonable decision at different time points.

In the future, a sensitive analysis of the proposed method should be performed. Meantime, the proposed method could be integrated with traditional multi-criteria decision-making methods such as TOPSIS, PROMETHEE, VIKOR, and DEMENTAL for selecting alternatives in the real environment. Additionally, a decision support system based on LCMDM should be developed to increase the usefulness of the proposed method.

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