Divisibility of Labor Supply and Involuntary Unemployment: A Perfect Competition Model

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Abstract

We show the existence of involuntary unemployment without assuming wage rigidity. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition with decreasing or constant returns to scale technology. Indivisibility of labor supply may be grounds for existing involuntary unemployment. However, we show that under some conditions there exists involuntary unemployment even when labor supply is divisible.

Keywords: involuntary unemployment, perfect competition, divisible labor supply

JEL Classifications: E12, E24.

1. Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements.

1. Nominal wage rate being set above the nominal reservation wage rate.

2. Employment level and economic welfare never improving by lowering the nominal wage rate.

Otaki(2009) assumes that wage rate is equal to the reservation wage rate at the equilibrium under the indivisibility of labor supply. In such a situation, however, unemployment is not involuntary. He has shown involuntary unemployment exists from using efficient wage bargaining according to McDonald and R. M. Solow (1981). This paper, however, does not depend on bargaining. Another reference to involuntary unemployment without wage rigidity is Umada (1997). He derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such a curve leads to

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involuntary unemployment without wage rigidity[1]. However, his model of firms' behavior is ad-hoc.

We consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition according to Otaki (2010, 2011, 2015) with decreasing or constant returns to scale technology, and show involuntary unemployment exists under divisibility of individual labor supply. Indivisibility of labor supply means that labor supply of each individual can be 1 or 0. On the other hand, if labor supply is divisible, it is a variable in $[0,1]$. As discussed by Otaki (2012, 2015), if labor supply is infinitely divisible, there exists no unemployment. However, if labor supply by each individual is not so small, there may exist involuntary unemployment even when labor supply is divisible. In this paper, the first element of Otaki's two elements of involuntary unemployment should be that labor supply of each individual is positive at the current real wage rate. In other papers, we have shown involuntary unemployment exists under perfect or monopolistic competition when labor supply by individuals is indivisible[2].

Section 2 analyzes consumers' utility maximization in an overlapping generations model with two periods. We consider labor supply by individuals as well as their consumptions; Section 3 we consider profit maximization of firms under perfect competition; and Section 4 shows the existence of involuntary unemployment when labor supply is divisible.

Schultz (1992) showed that involuntary unemployment does not arise in an overlapping generations model. His arguments depend on the real balance effect caused by a fall in the nominal wage rate on consumption of older generation consumers. This point is discussed in the Appendix.

2. Consumers

We consider a two-period (young and old) overlapping generations model under perfect competition according to Otaki (2010, 2011 and 2015). There is one factor of production, labor, and good that are produced under perfect competition. There is a continuum of firms. The volume of firms is one. Consumers are born at continuous density $[0,1] \times [0,1]$ in each period. Each consumer supplies l units of labor when they are young (the first period), $0 \leq l \leq 1$.

[1] Lavoie (2001) presented a similar analysis.

[2] In Tanaka(2020a) we have shown the existence of involuntary unemployment in an overlapping generations model with indivisible labor supply. In Tanaka(2020b) we analyzed the balanced budget multiplier in a static model (not overlapping generations model) under monopolistic competition with indivisible labor supply and have shown the existence of involuntary unemployment.

We use the following notations.

c_i: consumption of the good at period i, $i = 1,2$.

p_i: price of the good at period i, $i = 1,2$.

W: nominal wage rate.

Π: profits of firms that are equally distributed to each consumer.

l: labor supply of an individual.

L: employment (number of employed consumers (workers)).

L_f: population of labor or employment at the full-employment state.

$y(Ll)$: labor productivity, which is decreasing or constant for

"employment $\times$ labor supply (Ll)", $y(Ll) \geq 1$, $y' \leq 0$.

We drop the time argument of c_1 and c_2, which should be written as c_{1_t} and $c_{2_{t+1}}$, for simplicity.

We call Ll "net employment", and define the elasticity of the labor productivity for Ll as follows,

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}. \tag{1}$$

We assume that $-1 < \zeta \leq 0$, and ζ are constant. Decreasing (constant) returns to scale means $\zeta < 0$ ($\zeta = 0$). The output is $Lly(Ll)$. $\zeta > -1$ or $1 + \zeta > 0$ means that the output is increasing regarding Ll. If the good is produced under constant returns to scale technology, $\Pi = 0$.

The utility of a consumer of one generation over two periods is

$$U(c_1, c_2) = u(c_1, c_2) - G(l). \tag{2}$$

We assume that $u(c_1, c_2)$ is homogeneous of degree one (linearly homogeneous). $G(l)$ is a function of disutility of labor that is continuous, strictly increasing, differentiable and strictly convex, thus $G' > 0$, $G'' > 0$.

The budget constraint for an employed individual is

$$p_1 c_1 + p_2 c_2 = Wl + \Pi. \tag{3}$$

p_2 is the expectation of the price in period 2. In our model the good is produced by only labor, and there is no capital. Interest rate is assumed to be zero. The Lagrange function is

$$\mathcal{L} = u(c_1, c_2) - G(l) - \lambda[p_1 c_1 + p_2 c_2 - (Wl + \Pi)].$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial c_1} - \lambda p_1 = 0,\ \frac{\partial u}{\partial c_2} - \lambda p_2 = 0. \tag{4}$$

They are rewritten as

$$\frac{\partial u}{\partial c_1} c_1 = \lambda p_1 c_1,\ \frac{\partial u}{\partial c_2} c_2 = \lambda p_2 c_2. \tag{5}$$

Since $u(c_1, c_2)$ is homogeneous of degree one,

$$\frac{\partial u}{\partial c_1} c_1 + \frac{\partial u}{\partial c_2} c_2 = u(c_1, c_2) = \lambda(p_1 c_1 + p_2 c_2) = \lambda(Wl + \Pi),$$

and λ is a function of p_1 and p_2, and $\frac{1}{\lambda}$ is homogeneous of degree one regarding (p_1, p_2) because proportional increases in p_1 and p_2 reduce c_1 and c_2 at the same rate given $Wl + \Pi$. Then, we obtain the following indirect utility function.

$$V = \frac{1}{\varphi(p_1, p_2)}(Wl + \Pi) - G(l). \tag{6}$$

$\varphi(p_1, p_2)$ is a function of p_1 and p_2. It is positive, increasing regarding p_1 and p_2,[3] and homogeneous of degree one regarding (p_1, p_2). Therefore, the indirect utility function V is decreasing regarding p_1 and p_2, and homogeneous of degree zero regarding $(p_1, p_2, Wl + \Pi)$. Maximization of V concerning l implies

$$W = \varphi(p_1, p_2)G'(l). \tag{7}$$

Let $\rho = \frac{p_2}{p_1}$. From (7)

$$\omega = \frac{W}{p_1} = \varphi(1, \rho)G'(l). \tag{8}$$

ω is the real wage rate. If the value of ρ is given, l is obtained from (8) as a function of ω. Since $G'' > 0$, labor supply l increases with the real wage rate ω.

For an unemployed individual the budget constraint is $p_1 c_1 + p_2 c_2 = \Pi$, and his indirect utility function is

$$\frac{1}{\varphi(p_1, p_2)}\Pi. \tag{9}$$

Let

$$\alpha = \frac{p_1 c_1}{Wl + \Pi},\ 1 - \alpha = \frac{p_2 c_2}{Wl + \Pi}. \tag{10}$$

We have $0 < \alpha < 1$. Demand for good of each employed consumer of the younger

[3] λ is decreasing with respect to p_1 and p_2.

generation is

$$c_1 = \frac{\alpha(Wl + \Pi)}{p_1}. \tag{11}$$

Demand in the second period is

$$c_2 = \frac{(1-\alpha)(Wl + \Pi)}{p_2}. \tag{12}$$

For an unemployed consumer $c_1 = \frac{\alpha\Pi}{p_1}$, $c_2 = \frac{(1-\alpha)\Pi}{p_2}$. Let $\bar{c}_2$ be demand of an older generation consumer who works in the previous period, $\bar{l}$, $\bar{W}$, $\bar{\Pi}$ and $\bar{\alpha}$ are labor supply, the nominal wage rate, the profit and the value of α when he is young. Then

$$\bar{c}_2 = \frac{(1-\bar{\alpha})(\bar{W}\bar{l} + \bar{\Pi})}{p_1}. \tag{13}$$

$(1-\bar{\alpha})(\bar{W}\bar{l} + \bar{\Pi})$ is his savings carried over from his first period. For an older generation consumer who is unemployed in the previous period $\bar{c}_2 = \frac{(1-\bar{\alpha})\bar{\Pi}}{p_1}$. Total savings of the older generation consumers is M. Then, their demand for the good is

$$\frac{M}{p_1}. \tag{14}$$

The total demand for the good is

$$c = \frac{Y}{p_1}. \tag{15}$$

Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi) + G + M. \tag{16}$$

G is the government expenditure. Government expenditure and consumptions of younger and older generations constitute the national income (please see Otaki (2007), Otaki (2009), Otaki (2015)).

3. Firms

This section we consider firms' profit maximization behavior. Let x and z be the output and the net employment (employment $\times$ labor supply) of a firm. We have $x = y(z)z$ and

$$\zeta = \frac{y'}{\frac{y(z)}{z}}. \tag{17}$$

Thus,

$$\frac{dz}{dx} = \frac{1}{y(z) + y'z} = \frac{1}{(1+\zeta)y(z)}. \tag{18}$$

The profit of a firm is

$$\pi = p_1 x - \frac{x}{y(z)} W. \tag{19}$$

p_1 is given for each firm. The condition for profit maximization under perfect competition is

$$p_1 - \frac{y(z) - xy'\frac{dz}{dx}}{y(z)^2} W = p_1 - \frac{1 - y'z\frac{dz}{dx}}{y(z)} W = p_1 - \frac{1}{y(z) + y'z} W = p_1 - \frac{1}{(1+\zeta)y(z)} W = 0. \tag{20}$$

Therefore $p_1 = \frac{1}{(1+\zeta)y(z)} W$. This means the marginal cost pricing. Since at the equilibrium $x = c$ and $z = Ll$, we obtain

$$p_1 = \frac{1}{(1+\zeta)y(Ll)} W. \tag{21}$$

With decreasing (constant) returns to scale $-1 < \zeta < 0$ ($\zeta = 0$).

4. Main Results

4.1. Involuntary Unemployment

From (21) the real wage rate is

$$\omega = \frac{W}{p_1} = (1+\zeta)y(Ll). \tag{22}$$

Under decreasing (constant) returns to scale, since ζ is constant, ω is decreasing (constant) concerning Ll. (8) and (22) provide

$$(1+\zeta)y(Ll) = \varphi(1,\rho)G'(l). \tag{23}$$

From (23) labor supply of an individual l is obtained as a function of L. Denote it by $l(L)$. Since $G'' > 0$ (convex disutility of labor) and $y' \leq 0$ (decreasing or constant returns to scale), we have

$$\varphi(1,\rho)G''(l) - (1+\zeta)y'L > 0. \tag{24}$$

This guarantees that $l(L)$ is decreasing and $Ll(L)$ is strictly increasing concerning L because

$$\frac{dl(L)}{dL}=\frac{(1+\zeta)y'l(L)}{\varphi(1,\rho)G''(l)-(1+\zeta)y'L}\leq 0, \tag{25}$$

and

$$\frac{dLl(L)}{dL}=l(L)+L\frac{dl(L)}{dL}=\frac{\varphi(1,\rho)G''(l)l(L)}{\varphi(1,\rho)G''(l)-(1+\zeta)y'L}>0. \tag{26}$$

Then, the real wage rate ω decreases for L because $y'\leq 0$.

Alternatively, from (23) l is obtained as a function of Ll. Denote it by $l(Ll)$. Then,

$$\frac{dl(Ll)}{d(Ll)}=\frac{(1+\zeta)y'}{\varphi(1,\rho)G''}\leq 0. \tag{27}$$

The (nominal) aggregate supply of the good is equal to

$$WLl+\Pi=p_1Lly(Ll). \tag{28}$$

Ll is an abbreviation of $Ll(L)$ or $Ll(Ll)$. The (nominal) aggregate demand is

$$\alpha(WLl+\Pi)+G+M=\alpha p_1Lly(Ll)+G+M. \tag{29}$$

Since they are equal,

$$p_1Lly(Ll)=\alpha p_1Lly(Ll)+G+M,\ \text{or}\quad p_1Lly(Ll)=\frac{G+M}{1-\alpha}. \tag{30}$$

In real terms[4]

$$Lly(Ll)=\frac{1}{1-\alpha}(g+m),\ \text{or}\quad Ll=\frac{1}{(1-\alpha)y(Ll)}(g+m), \tag{31}$$

where

$$g=\frac{G}{p_1},\qquad m=\frac{M}{p_1}.$$

(31) means that the net employment Ll is determined by $g+m$. $Lly(Ll)$ is strictly increasing regarding Ll because

$$\frac{d(Lly(Ll))}{d(Ll)}=y(Ll)+Lly'=y(Ll)\left(1+\frac{Lly'}{y(Ll)}\right)=y(Ll)(1+\zeta)>0. \tag{32}$$

Therefore, there exists the unique value of Ll that satisfies (31) given $g+m$. It is strictly increasing regarding $g+m$. From (23) we obtain the value of $l(Ll)$, and the value of L is determined by $L=\frac{Ll}{l(Ll)}$. Ll can not be larger than $L_fl(L_f)$ but may be strictly smaller than $L_fl(L_f)$. Then, there exists *involuntary umemployment*, that is, $L<L_f$ because Ll is

[4] $\frac{1}{1-\alpha}$ is a multiplier.

increasing regarding L.

(31) means

$$Lly(Ll) = \frac{G+M}{(1-\alpha)p_1}. \tag{33}$$

This is the aggregate demand function given $G+M$. $Lly(Ll)$ is constant given $g+m$. It is increasing regarding $G+M$ given p_1, and decreasing regarding p_1 given $G+M$. Since $Lly(Ll)$ is increasing regarding Ll, (33) means that the net employment Ll is increasing for $G+M$ given p_1 and decreasing for p_1 given $G+M$.

4.2. Summary of Discussions

The real aggregate demand and net employmnet Ll are determined by the real value of $g+m$. Labor supply of each individual is determined by Ll according to (23), and the employment L is determined by

$$L = \frac{Ll}{l(Ll)}. \tag{34}$$

Employment smaller than the population of labor suggests involuntary unemployment. Real wage rate is determined by Ll according to (22). There exists no mechanism to reduce involuntary unemployment unless $g+m$ is increased.

However, if

$$(1+\zeta)y(Ll) > \varphi(1,\rho)G'(l) \quad \text{for} \quad \text{any} \quad 0 < l < 1, \text{given} \quad L,$$

individuals choose $l = 1$, and then the labor supply is indivisible.

However, if

$$\lim_{Ll \to 0}(1+\zeta)y(Ll) < \varphi(1,\rho)G'(0), \tag{35}$$

individuals choose $l = 0$. However, if $G'(0)$ is sufficiently small, $l > 0$.

Involuntary unemployment occurs when aggregate demand for the good is insufficient. Firms determine the number of employed workers needed to meet the demand for the good. If demand for the good is insufficient, the number of employed workers may be smaller than the population. Then, there exists involuntary unemployment. Under decreasing returns to scale the real wage rate is decreasing regarding the output or employment. It may affect the individual labor supply. However, labor demand or employment (the number of employed workers) is not determined by the real wage rate. Under constant returns to scale the real wage rate is constant. Labor demand is determined by the aggregate demand for the good.

4.3. Comment on The Nominal Wage Rate

The reduction of the nominal wage rate induces a proportionate reduction of the price even when there exists involuntary unemployment, and it does not rescue involuntary unemployment (please see Otaki (2016, Ch. 2)[5].

In this section's model no mechanism determines the nominal wage rate. When the nominal value of $G+M$ increases, nominal aggregate demand and supply increase. If the nominal wage rate rises, price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G+M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G+M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[6].

4.4. Full-employment Case

If $L=L_f$, full-employment is realized. Then, (31) is written as

$$L_f l(L_f) y(L_f l(L_f)) = \frac{1}{1-\alpha}(g+m). \tag{36}$$

$l(L_f)$ is obtained from

$$(1+\zeta)y(L_f l) = \varphi(1,\rho)G'(l).$$

$L_f l(L_f) > Ll(L)$ for any $L < L_f$ because $Ll(L)$ is strictly increasing with respect to L. Since $L_f l(L_f)$ is constant, (36) is an identity not an equation. Conversely, (31) is an equation not an identity. (36) should be written as

$$\frac{1}{1-\alpha}(g+m) \equiv L_f \ l(L_f) y(L_f \ l(L_f)). \tag{37}$$

This defines the value of $g+m$ which realizes the full-employment state.

From (37) we have

$$p_1 = \frac{1}{(1-\alpha)L_f l(L_f) y(L_f l(L_f))}(G+M), \tag{38}$$

[5] However, there is room for improvement of employment by the real balance effect if the nominal value of consumption by the older generation consumers is maintained. About this point please see the Appendix.

[6] As mentioned in the Introduction, Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining. The arguments of this paper, however, do not depend on bargaining.

where

$$g = \frac{G}{p_1}, \qquad m = \frac{M}{p_1}.$$

Therefore, the price level p_1 is determined by $G + M$, which is the sum of nominal values of the government expenditure and consumption by the older generation. Also the nominal wage rate is determined by

$$W = (1 + \zeta)y(L_f l(L_f))p_1. \tag{39}$$

4.5. Government Expenditure to Reduce Unemployment

Now we assume that the government collects a lump-sum tax T from younger generation consumers. The aggregate demand and the aggregate supply are

$$\alpha(WLl + \Pi - T) + G + M = \alpha(p_1 Lly(Ll) - T) + G + M = p_1 Lly(Ll). \tag{40}$$

If Ll is a steady-state value with constant price and output, the savings of the younger generation must be equal to M. Thus,

$$(1 - \alpha)(p_1 Lly(Ll) - T) = G - T + M = M. \tag{41}$$

This means $G - T = 0$. Supposing that full-employment is realized by the government expenditure G' with the tax T' under constant price. Then, we get

$$(1 - \alpha)(p_1 L_f ly(L_f l) - T') = G' - T' + M. \tag{42}$$

Assume $p_1 L_f ly(L_f l) - T' > p_1 Lly(Ll) - T$, that is, the realization of full employment will increase consumers' disposable income. Then, from (41), (42) is $G' - T' > 0$. Therefore, budget deficits are necessary to move from the presence of involuntary unemployment to full-employment.

A Simple Example

Assume $M = 0$ and $T = 0$ in (40) with $L = L_f$. Then,

$$\alpha p_1 L_f ly\left(L_f l\right) + G = p_1 L_f ly(L_f l). \tag{43}$$

This means

$$G = (1 - \alpha)P_1 L_f ly(L_f l). \tag{44}$$

This is the government expenditure necessary to achieve full-employment when the older genaration's savings is zero, and equals the younger generation's savings. Denote it by M'.

Let G' and T' be the government expenditure and the tax in the next period. The following relation holds under constant price.

$$p_1 L_f ly(L_f l) = \alpha\left[p_1 L_f ly\left(L_f l\right) - T'\right] + G' + M'. \tag{45}$$

To maintain full-employment with $T' = 0$, the younger generation's savings must be M'. Therefore, we need

$$(1-\alpha)P_1 Lly(Ll) = G' + M' = M'. \tag{46}$$

This means

$$G' = 0. \tag{47}$$

4.6. Graphical Representation

The output of good $Lly(Ll)$ is an increasing function of the net employment Ll as shown in (32). (21) means that the price of the good is an increasing function of Ll given W due to decreasing returns to scale $y'(Ll) \leq 0$. Therefore, the price of the good is an increasing function of the output $Lly(Ll)$ given W. Denote this as

$$p_1 = \Psi(Lly(Ll)), \qquad \text{with} \quad \Psi' \geq 0. \tag{48}$$

Figure 1 presents a graphical representation of arguments. The line (48) represents Eq. (48). If the production process is constant returns to scale, the price is constant, and the line (48) is horizontal. The curve (33) represents Eq. (33). (33) expresses that the aggregate demand for the good is decreasing for p_1 given $G + M$ which is the sum of the nominal values of the government expenditure and the older generation consumers' savings. The intersection point represents the equilibrium. The equilibrium value of $Lly(Ll)$ may be smaller than $L_f ly(L_f l)$.

The thick curve represents Eq. (33) when the government expenditure G increases, which increases the equilibrium value of $Lly(Ll)$.

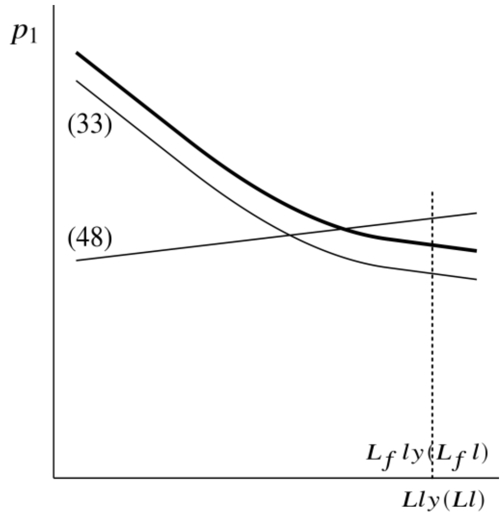

Figure 1: Equilibrium with Involuntary Unemployment

5. Concluding Remark

We have examined the existence of involuntary umemployment using a simple perfect competition model with divisible individual labor supply under decreasing or constant returns to scale technology. It seems to be possible to extend our analyses in this paper to monopolistic competition with increasing or constant returns to scale technology without changing main conclusions.

Appendix

As mentioned in Introduction Schultz (1992) showed the impossibility of involuntary unemployment in an overlapping generations model. His arguments depend on the real balance effect caused by a fall in the nominal wage rate on consumption of the older generation consumers. It is stated in Schultz(1992, p. 69) that:

In our model the presence of an old generation makes the real balance effect so strong that involuntary unemployment is excluded.

As noted in footnote 5, the real balance effect caused by falling nominal wage rate and prices may reduce unemployment. We consider a three generations overlapping generations model with pay-as-you go pension to explore the possibility of avoiding the real balance effect. Consider the following economy.

1. Each consumer lives three periods, Period 0 (childhood period), Period 1 (younger period) and Period 2 (older period). There are three generations, childhood, younger and older generations. In Period 0 the consumers consume D units of the good by borrowing money from consumers in the previous generation. D is constant and the debts must be repaid in Period 1.

2. In Period 1 the consumers are employed by firms or unemployed. Unemployed consumers can not repay their debts. Therefore, each unemployed consumer receives unemployment benefit that equals his debt in Period 0. The unemployment benefits are covered by taxes on employed younger consumers who also pay taxes for pay-as-you-go pensions for older generation consumers.

3. In Period 2 consumption of retired consumers is financed by their savings and pensions.

4. Since consumption in Period 0 of each consumer is constant, he determines his consumptions in Period 1 and Period 2 when Period 1 starts if he is employed or unemployed.

We use the following notations.

D: constant consumption of an individual in the childhood period. It is constant.

$\widehat{D}$: consumption of a next generation consumer in the childhood period.

R: unemployment benefit for an unemployed individual. $R = D$.

Θ: tax payment by an employed individual for the unemployment benefit.

Q: pay-as-you-go pension for an individual of the older generation.

$\hat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.

Ψ: tax payment by an employed individual for the pay-as-you-go pension for the older generation consumers.

The following relationships hold.

$$L\Theta = (L_f - L)D, \qquad L\Psi = L_f Q. \tag{A.1}$$

The budget constraint of an employed consumer is

$$p_1c_1 + p_2c_2 = Wl + \Pi - D - \Theta - \Psi + \hat{Q}, \tag{A.2}$$

and that of an unemployed consumer is

$$p_1c_1 + p_2c_2 = \Pi + \hat{Q}. \tag{A.3}$$

The analysis that follows is similar. The (nominal) effective demand is

$$\begin{aligned} Y &= \alpha\big(WLl + L_f\Pi - L(D + \Theta) - L\Psi + L_f\hat{Q}\big) + G + M + \widehat{D} \\ &= \alpha(WLl + L_f\Pi - L_fD - L_fQ + L_f\hat{Q}) + G + M + \widehat{D}. \end{aligned} \tag{A.4}$$

M is the total consumption of the older generation consumers which are financed by their savings and pay-as-you go pensions. Their net savings is

$$M - L_fQ. \tag{A.5}$$

The Effects of Falling in The Nominal Wage Rate

If the nominal wage rate falls and the price of the good proportionately falls, we can assume that the real values of the following variables are maintained.

$$\Pi,\ Q,\ \hat{Q},\ G \quad \text{and} \quad \widehat{D}.$$

On the other hand, the nominal values of D and $M - L_fQ$ are not changed. Therefore, whether a fall in the nominal wage rate increases or decreases the effective demand depends on whether

$$M - \alpha L_fD - L_fQ. \tag{A.6}$$

is positive or negative. If it is negative, a fall in the nominal wage rate decreases the effective demand and increases involuntary unemployment.

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