Financial Deepening and Monetary Growth with Endogenous Liquidity Constraints

Ming-fu Shaw
Department of Economics, National Cheng Chi University, Taiwan

Ching-chong Lai*
Institute of Economics, Academia Sinica, Taiwan
Department of Economics, National Cheng Chi University, Taiwan
Department of Economics, Feng Chia University, Taiwan

This paper sets up a monetary growth model with an endogenous liquidity constraint on consumption, and uses it to analyze the economy’s long-run response to money growth. We find that a rise in the rate of money growth will increase the consumer’s financial deepening, thereby leading to a reduction in the liquidity constraint on consumption and a rise in the consumption-capital ratio. With such an adjustment, the representative household can borrow more and raise its consumption. As a consequence, a rise in the rate of money growth will deter the savings rate and the economic growth rate.

Keywords: endogenous liquidity constraints, financial deepening, endogenous growth

JEL classification: E52, O42
1 Introduction

The effect of money growth on capital formation has been the central issue in the money and growth literature; see, for example, Tobin (1965), Sidrauski (1967), and Stockman (1981). In their survey paper, Gylfason and Herbertsson (2001, p. 408) point out that a common observation in empirical studies is that there exists a negative relationship between the inflation rate and economic growth in the long run. Based on this finding, some studies have developed distinct theoretical models to fit the empirical observations. Within the literature, Marquis and Reffett (1991) and Mino (1997) introduce money to a two-sector Lucas (1988) model through the cash-in-advance constraint. They conclude that the inflation tax lowers the balanced growth rate unless only consumption is liquidity constrained and the household’s labor supply is fixed inelastically. Gomme (1993) further finds that, if labor supply is endogenously chosen, a rise in monetary growth reduces the long-run growth rate even if only consumption is liquidity constrained. Chu and Lai (2013) develop a monetary R&D-based endogenous growth model in which innovation serves as the main engine of economic growth. Their analysis reveals that a rise in the rate of monetary growth leads to a detrimental effect on the balanced growth rate if the elasticity of substitution between consumption and the real money balance is less than unity.

In the context of a credit constraint, the higher liquidity constraint on consumption implies that the household cannot borrow the desired amount, and the savings rate will be higher than that in the presence of perfect credit markets. Jappelli and Pagano (1994) show that a household’s higher credit constraint tends to raise its savings rate and thus promote economic growth. Using cross-country data, Liu and Woo (1994) also find that there may exist a positive relationship between credit market imperfections and savings. Additionally, empirical studies indicate that both the consumer credit market and the producer credit market can be affected by financial innovation and deepening in an economy’s development. Miles (1992) shows that financial deregulation is a key determinant of the sharp decline in the U.K. savings rate in the 1980s. Vaidyanathan (1993) finds a negative relationship
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between the household’s credit constraint and financial deepening.\(^1\) As is evident in these studies, the consumer’s liquidity constraint is closely related to the development of credit markets.

In the context of monetary growth, however, less attention has been devoted to studying whether the linkage between the consumer’s liquidity constraint and the development of credit markets will govern the economic growth rate. In view of this fact, by embedding the \(Ak\) type of endogenous growth into the continuous time version of Stockman’s (1981) framework, this paper makes a new attempt to set out a theoretical model, and uses it to provide an alternative vehicle for the empirical linkage between money growth and economic growth. In a departure from the existing literature that focuses on the households’ demand for credit, this article highlights the role of the supply of credit to households.

The rest of the paper proceeds as follows. The structure of the economic system with endogenous liquidity constrained on consumption characterized by money and endogenous growth is outlined in Section 2. The economy’s long-run responses to a rise in monetary growth are examined in Section 3. Finally, Section 4 concludes by summarizing the major findings.

2 The Model

The model we use is a modified continuous time version of Stockman’s (1981) framework. The economy consists of a representative household and a government. The monetary authorities set the nominal money supply to grow at a fixed rate.

We assume that a fraction \(\alpha\) of the consumption good is constrained with the holding of cash and the fraction is a decreasing function of financial deepening.\(^2\) For instance, a higher trade volume made by using credit cards is associated with a relatively lower trade volume transacted by cash. Financial deepening is measured by a real financial service-production ratio \(f/y\), where \(f\) denotes real financial service and \(y\) denotes output. Consequently, in line with Vaidyanathan (1993) and

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\(^1\)The same interaction between the credit constraint and financial development can be found in Palivos \textit{et al.} (1993), although the liquidity constraint on investment they propose is that exogenous credit enhancement will reduce the liquidity constraint on investment.

\(^2\)A similar specification can be found in Palivos \textit{et al.} (1993), but they focus on the liquidity constraint on investment.
Jappelli and Pagano (1994), the liquidity constraint ratio, \( \alpha(\cdot) \), is specified as 
\[ \alpha \equiv \alpha(f/y) \]  
with \( \alpha' < 0 \) and \( \alpha'' > 0 \).

Subject to the intertemporal budget constraint and the liquidity constraint ratio, 
the representative household’s optimal decision is to choose real consumption \( c \), 
real investment \( i \), real capital \( k \), real financial service \( f \), and real money 
balances, \( m = M/P \), where \( M \) is the nominal money balance and \( P \) is the 
price level. That is, the representative household’s plans are obtained by solving the 
following intertemporal optimization problem:

\[
\begin{align*}
\text{Max}_{(c,i,k,f)} & \int_0^\infty \ln c \cdot e^{\rho t} dt, \\
\text{s.t.} & \ y = Ak, \\
& \dot{m} = (1-\phi)y - c - i - \pi m + \tau, \\
& \dot{k} = i, \\
& \dot{f} = \phi y, \\
& \alpha(f/y) \cdot c \leq m, \ \alpha' < 0, \ \alpha'' > 0, \\
& k(0) = k_0, \ m(0) = m_0, \ f(0) = f_0,
\end{align*}
\]

where \( \rho \) is the constant rate of time preference, \( \pi \) is the inflation rate, and \( \tau \) is 
real transfers from the government. Following Pagano (1993), \( \phi \) can be viewed as 
a commission which is absorbed by financial intermediaries in order to accumulate 
financial services.

Equation (1) states that the objective of the household is to maximize the 
discounted sum of future instantaneous utilities. Equation (2) indicates that the 
production function takes the ‘\( Ak \)’ technology form. As claimed by Lucas (1988), 
\( k \) can be viewed as a composite of physical and human capital. Equations (3) and 
(4) are respectively the familiar flow budget constraint and the physical capital 
accumulation constraint with \( M \) and \( k \) being given at their initial values \( M_0 \) and 
\( k_0 \). Assume that financial intermediaries charge a fraction \( \phi \) of output 
production, and invest it to promote financial services. Equation (5) describes the 
process of financial service accumulation. Equation (6) indicates that the fraction 
\( \alpha \) of the consumption good is purchased with money. The fraction \( 1 - \alpha \) of the 

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\(^1\)Our main results hold if the instantaneous utility function \( U \) is specified as 
\[ U = (c^{1-\sigma})/(1-\sigma), \]  
where \( \sigma \) is the inverse of the elasticity of intertemporal substitution. When \( \sigma = 1 \), the instantaneous 
utility function degenerates to the form \( U = \ln c \).
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Consumption good, on the other hand, is purchased with credit, with the fraction \( \alpha \) being assumed to depend on an endogenous credit enhancement index, i.e., financial deepening \( f/y \). Furthermore, it is plausibly specified that the higher degree of financial deepening is associated with a lower liquidity constraint ratio, that is, \( \alpha' < 0 \).

The optimum conditions necessary for the representative household are:

\[
\frac{1}{c} = \lambda + \alpha \gamma, \quad \lambda = q, \quad -\lambda + \rho \lambda = -\lambda \pi + \gamma, \quad -\dot{q} + \rho q = \lambda (1 - \phi) A + v \phi A + \gamma \alpha' \cdot \frac{f}{Ak} c, \quad \dot{v} + \rho v = -\gamma \alpha' \cdot \frac{1}{A} c, \quad m = \alpha (f/Ak) \cdot c, \quad \lim_{t \to \infty} \lambda m \cdot e^{\pi t} = 0, \quad \lim_{t \to \infty} q k \cdot e^{\pi t} = 0, \quad \lim_{t \to \infty} v f \cdot e^{\pi t} = 0, \quad \lim_{t \to \infty} = \frac{\lambda}{m} - \frac{\lambda}{\alpha} - \frac{\lambda}{\alpha} \cdot \frac{1}{\alpha} \cdot \frac{f}{Ak} m. \quad (15)
\]

where \( \lambda \) is the shadow value of wealth, \( q \) is the shadow value of the capital stock, \( v \) is the shadow value of financial services, and \( \gamma \) is the Lagrange multiplier corresponding to the liquidity constraint (6). Equation (7) implies that the marginal utility of consumption equals the sum of the marginal utility of wealth and the \( \alpha \) proportion of the marginal utility of holding money. Equation (13) represents the transversality conditions of \( m, k, \) and \( f, \) and ensures that the household’s intertemporal budget constraint is met.

Combining (7) with (12) gives:

\[
\gamma = \frac{1}{m} - \frac{\lambda}{\alpha}. \quad (14)
\]

Combining equations (8)–(10) and (14) together, we can infer the inflation tax \( \pi \):

\[
\pi = \frac{1}{\lambda m} - \frac{1}{\alpha} \cdot \frac{1}{\alpha} \cdot \frac{1}{\alpha} \cdot \frac{f}{Ak} m. \quad (15)
\]
Assume that the government keeps the money growth rate at a constant rate \( \mu \) (\( = M/M \)) and distributes seigniorage to the representative household as a transfer payment in a lump-sum manner. The government’s flow budget constraint can thus be written as:

\[
\dot{m} = \tau - \pi m. 
\]

Moreover, the law of motion governing real cash balances and the transfer payment of the government can be represented as:

\[
\frac{\dot{m}}{m} = \mu - \pi, \quad (16)
\]

\[
\tau = \mu \tau. \quad (17)
\]

Combining equations (3), (4), (16), and (17) together, the economy’s consolidated budget constraint can be expressed as:

\[
\dot{k} = (1 - \phi)A k - c. \quad (18)
\]

It follows from equations (10) and (18) that the transversality condition \( \lim_{t \to \infty} qk \cdot e^{-\rho t} = 0 \) reported in equation (13) will be met if and only if:

\[
\frac{1}{\alpha'} \left( \frac{m}{k} \right) + \left( \frac{\nu}{\lambda} \right) s + \left( \frac{1}{\lambda m} - \frac{1}{\alpha'} \right) (\alpha')' \left( \frac{f}{A} \right) \left( \frac{m}{k} \right) > 0, \quad (19)
\]

where the superscript “*” stands for the stationary value of relevant variables and \( \alpha' \) equals \( \alpha((f/A)') \). Equation (19) also ensures that the transversality conditions \( \lim_{t \to \infty} \lambda m \cdot e^{-\rho t} = 0 \) and \( \lim_{t \to \infty} vf \cdot e^{-\rho t} = 0 \) in equation (13) are met because \( m, f, \) and \( k \) have a common growth rate. This common growth rate is defined as \( g' = \dot{y}/y = k/k = \dot{m}/m = f/f = -\delta/\lambda = -\dot{v}/\nu. \)

Following Futagami et al. (1993) and Barro and Sala-i-Martin (2004), define \( x = f/k, \ s = m/k, \ z = 1/\lambda m, \) and \( \delta = v/\lambda. \) Combining equations (2), (12), (14), and (15) together, we can then rewrite equations (5), (9), (11), (16), and (18) as follows:

\[
\frac{\dot{x}}{x} = \frac{\phi 4}{x} - (1 - \phi) A + \frac{s}{\alpha(x/A)}, \quad (20)
\]

\footnote{See the Appendix for a detailed proof.}
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\[ \frac{\dot{s}}{s} = \mu - z + \frac{1 + s}{\alpha(x/A)} + \delta \phi A + \left( z - \frac{1}{\alpha(x/A)} \right) \cdot \frac{\alpha'(x/A)}{\alpha(x/A)} \cdot \frac{xs}{A}, \]  
\[ \frac{\dot{z}}{z} = z - \frac{1}{\alpha(x/A)} - \mu - \rho, \]  
\[ \frac{\dot{\delta}}{\delta} = (1 - \phi)A + \delta \phi A + \left( x + \frac{1}{\delta} \right) \cdot \left( z - \frac{1}{\alpha(x/A)} \right) \cdot \frac{\alpha'(x/A)}{\alpha(x/A)} \cdot \frac{s}{A}. \]

At the balanced growth equilibrium, the economy is characterized by \( \dot{x} = \dot{s} = \dot{z} = \dot{\delta} = 0 \) and \( x, s, z, \) and \( \delta \) are at their stationary values, namely, \( x^*, s^*, z^* \) and \( \delta^* \). Based on equations (20)–(23), \( x^*, s^*, z^* \) and \( \delta^* \) are determined by:

\[ \frac{\phi A}{x^*} - (1 - \phi)A + \frac{s^*}{\alpha(x^*/A)} = 0, \]  
\[ \mu - z^* + \frac{1 + s^*}{\alpha(x^*/A)} + \delta \phi A + \left( z^* - \frac{1}{\alpha(x^*/A)} \right) \cdot \frac{\alpha'(x^*/A)}{\alpha(x^*/A)} \cdot \frac{x^* s^*}{A} = 0, \]  
\[ z^* - \frac{1}{\alpha(x^*/A)} - \mu - \rho = 0, \]  
\[ (1 - \phi)A + \delta \phi A + \left( x^* + \frac{1}{\delta} \right) \cdot \left( z^* - \frac{1}{\alpha(x^*/A)} \right) \cdot \frac{\alpha'(x^*/A)}{\alpha(x^*/A)} \cdot \frac{s^*}{A} = 0. \]

It is clear from equations (21a) and (22a) that equation (19) is met.

We first have to explain the implication in equations (5) and (6): \( g^* = \phi A/x^* \) and \( \alpha'(x^*/A) < 0 \). These two relationships indicate that a rise in the consumer’s financial deepening, \( x^*/A \), will lower the balanced growth rate and impose less of a liquidity constraint on consumption. Consequently, the relationship between the balanced growth rate and the liquidity constraint on consumption, as put forth by Jappelli and Pagano (1994), is positive. It should be mentioned that the positive linkage between liquidity constraints and the growth rate is quite different from that of Bencivenga and Smith (1991) and Greenwood and Jovanovic (1990). In those studies, financial intermediation promotes growth, because it increases the rate of return on capital via a more efficient allocation of credit to investment and production (Pagano, 1993). However, our model emphasizes the channel of financial development on consumption.
3 Monetary Growth and Liquidity Constraints

This section will examine the effect of a permanent increase in the rate of money growth. By linearizing equations (20a)–(23a) around the steady-state equilibrium, we have:

\[
\begin{pmatrix}
a_{11} & a_{12} & 0 & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & 0 & a_{33} & 0 \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\begin{pmatrix}
dx^* \\
d\delta x^* \\
dz^* \\
d\delta z^*
\end{pmatrix}
=
\begin{pmatrix}
0 \\
-s^* \\
z^* \\
0
\end{pmatrix}
d\mu ,
\] (24)

where

\[
a_{i1} = -(\phi A/x^*) - (\alpha'/\alpha^*) (s^*/A), \\
a_{12} = x^*/\alpha , \\
a_{21} = s^*/\delta^* \Omega - (\alpha'/\alpha^*) (s^*/A) - (\alpha'/\alpha^*) (\delta^* x^* \Omega/\mu + \rho)(1/A) - [(\alpha'/\alpha^*) - (\alpha'/\alpha^*)](\delta^* x^* \Omega/4) , \\
\Omega = \rho + (\phi A/x^*) , \\
a_{22} = s^* [(1/\alpha) - (\delta^* x^* \Omega/s^*)] , \\
a_{23} = s^* [1 - (\delta^* x^* \Omega/\mu + \rho)] , \\
a_{24} = s^*/\phi A , \\
a_{31} = (z^*/A)(\alpha'/\alpha^*) , \\
a_{33} = z^* , \\
a_{34} = \delta^* [-(\delta^* x^* \Omega/\mu + \rho)] , \\
a_{41} = -\delta^* (\delta^* x^* \Omega + \Omega/\mu + \rho) , \\
a_{42} = \delta^* [-(\delta^* x^* \Omega/x^*)] , \\
a_{43} = \delta^* [\phi A + (\Omega/\delta^*)] .
\]

As claimed in the literature on dynamic rational expectations models, including Buiter (1984) and Turnovsky (1995), the dynamic system has a unique perfect foresight equilibrium if the number of unstable roots equals the number of jump variables. Given that the dynamic system reported in equations (20)–(23) has three jump variables, \( s, z \), and \( \delta \), in what follows we impose the determinant \( \Delta < 0 \) to ensure such a unique perfect foresight equilibrium. The determinant \( \Delta \) can be expressed as:

\[
\Delta = x^* s^* z^* \delta^* \cdot \frac{\rho}{\alpha} \Omega \Psi .
\] (25)

With \( \Delta < 0 \), this further implies that \( \Psi = 1 + (\alpha'/\alpha^*) (x^*/A) + (\phi A/x^*) [(1/\Omega) - (\alpha'/\alpha^*) (s^*/\delta^* x^*)] < 0 \) should be satisfied. The inequality \( \Psi < 0 \) thus gives the lower bound for \( \alpha^* \). \(^7\)

From equation (24) we can derive the following steady-state relationship:

\(^7\)The lower bound for \( \alpha^* \) is \( \alpha^* > -\alpha' (Axf(x^*))(1/\Omega) - (\alpha'/\alpha^*) (s^*/\delta^* x^*) \).
Equations (26) and (27) imply that a rise in the rate of money growth $\mu$ increases the financial service-capital ratio $\frac{f}{y}$ and the consumption-capital ratio $\frac{c}{y}$. The degree of the consumer’s financial deepening $\frac{y}{y}$ thus increases.

Equations (28) and (29) tell us that a rise in money growth has a negative impact on both the liquidity constraint on consumption $\alpha$ and the balanced growth rate $g$.

The above long-run results can be intuitively explained as follows. A rise in the rate of money growth increases the financial service-capital ratio and financial deepening on consumption. A rise in financial deepening on consumption, in turn, reduces the ratio of the borrowing constraint on consumption and hence increases the consumption-capital ratio. The representative household then can borrow more and aggregate savings will be lowered in response, thereby resulting in a decline in both the savings rate and the balanced growth rate. This result is consistent with the empirical evidence proposed by Roubini and Sala-i-Martin (1992), De Gregorio (1993), and Gylfason and Herbertsson (2001). Our results stand in contrast to the literature on monetary endogenous growth with cash in advance; for example, Marquis and Reffett (1991) and Mino (1997). In those studies, with only consumption being liquidity constrained, money is superneutral with respect to monetary growth in the long run. In this paper we find that, via an endogenous liquidity constraint, the long-run money superneutrality is not valid even if only consumption is liquidity constrained.

4 Concluding Remarks

This paper employs an $Ak$ type endogenous growth model with an endogenous liquidity constraint on consumption to examine the effect of money growth on the
balanced growth rate. The previous studies on monetary endogenous growth with the cash-in-advance constraint, such as Marquis and Reffett (1991), Mino (1997), and Chang et al. (2000), impose the restriction that the fractions of consumption goods and investment goods financed by money balances are exogenously given constants. In departing from these studies, this paper develops an *endogenous* liquidity constraint related to the degree of financial deepening, and finds that a rise in the money growth rate leads to a reduction in the balanced growth rate even if only consumption goods are liquidity constrained. As is evident, the *endogenous* liquidity constraint can serve as a plausible vehicle to explain the negative linkage between money growth and the economic growth rate put forth by Roubini and Sala-i-Martin (1992), De Gregorio (1993), and Gylfason and Herbertsson (2001).

**Appendix**

Given equations (8), (12), and (14), from equations (10) and (18) we have:

\[
\lambda(t) = \lambda_0 \cdot \exp \left\{ \alpha - \int_0^t \left[ (1-\phi) A + \frac{v}{\lambda} \phi A + \left( \frac{1}{\lambda m} - \frac{1}{\alpha} \right) \frac{f}{\alpha} \cdot \frac{m}{A k} \right] d\zeta \right\}, \tag{A1}
\]

\[
k(t) = k_0 \cdot \exp \left\{ \int_0^t \left[ (1-\phi) A - \frac{1}{\alpha} \frac{m}{k} \right] d\zeta \right\}. \tag{A2}
\]

Furthermore, the transversality condition \( \lim_{t \to \infty} qk \cdot e^{\rho t} = 0 \) in equation (13) can be rewritten as:

\[
\lim_{t \to \infty} qk \cdot e^{\rho t} = \lambda_0 k_0 \cdot \lim_{t \to \infty} \exp \left\{ -\int_0^t \left[ \frac{1}{\alpha} \frac{m}{k} + \frac{v}{\lambda} \phi A + \left( \frac{1}{\lambda m} - \frac{1}{\alpha} \right) \frac{f}{\alpha} \cdot \frac{m}{A k} \right] d\zeta \right\} = 0.
\]

In order to ensure that the household’s intertemporal budget constraint is satisfied, given that \( \lambda \to \lambda' \) and \( v \to v' \) exponentially along the transitional adjustment path, it follows from the above equation that \( \lim_{t \to \infty} qk \cdot e^{\rho t} = 0 \) will be met if and only if:

\[
\frac{1}{\alpha} \left( \frac{m}{k} \right)' + \left( \frac{v}{\lambda} \right) \phi A + \left( \frac{1}{\lambda m} - \frac{1}{\alpha} \right) \frac{f}{\alpha} \cdot \frac{1}{A k} \left( \frac{m}{k} \right)' > 0. \tag{A3}
\]

Given that the economy is characterized by \( \dot{x} = \dot{s} = \dot{z} = \dot{\delta} = 0 \) at the balanced
growth equilibrium, it is quite clear from equations (20)-(23) that 
\( g' = g' = g' = g' \) should hold. With the relation 
\( g' = g' = g' = g' \), the 
transversality conditions stated in equation (13), 
\( \lim_{t \to \infty} \lambda t = 0 \), 
\( \lim_{t \to \infty} v t = 0 \), are met.

References


