Divisionalization, Franchising, or Mixing: A Market Competition Perspective

Chi-Chih Lin*
Department of Economics, Fo Guang University, Taiwan

This paper considers strategic delegation for analyzing the optimal retail-organizational form for a firm. Our results indicate that expansion by a mix of divisions and franchises is possible, provided that there exists hierarchical conflict, the market is medium-sized, and the products are imperfect substitutes. The sales maximization for divisionalization plays an important role in the choice of retail-organizational form. This stems from the fact that the equilibrium outcome of profit delegation is more competitive than that of market share delegation, which itself is more competitive than that of sales delegation, where firms rely on expansion by a mix of divisions and franchises. Consequently, firms do not franchise to avoid a cost disadvantage, provided that top managers are guided by profit maximization. When the objective of each top manager is market share maximization, all top managers expect their rivals will retaliate, and this reduces the temptation to be aggressive in the first place. Accordingly, firms do not divisionalize to relax interbrand competition.

Keywords: divisionalization, franchising, market sale, product differentiation

JEL classification: D43, L10, L20

1 Introduction

It has long been argued that the objective of a firm might be different from profit maximization.1 Baumol (1958) indicates that managers might be guided to maximize total revenues rather than profits. Hall (1967) and Lackman and Craycraft (1974) cite evidence to empirically support the sales maximization hypothesis. Furthermore,
Zabojnik (1998) argues that “it is a well-documented empirical observation that the salaries of top managers increase with firm sales,” and he suggests that this relationship is causal rather than a reflection of larger firms. In other words, it is a widespread scenario in all industries.

If the ultimate goal of a manager is market power, then he may be more concerned with market share. Jansen et al. (2007) cite empirical findings of several surveys to argue that market share maximization often is important. Ritz (2008) argues that “market share objectives are prominent in many industries, especially where managers pay much attention to league table rankings,” and he gives the automotive and investment banking industries as two good examples. Wang and Wang (2008) suggest that firms may introduce brand proliferation to increase their market share; they indicate that “the practice of brand proliferation is visibly evident in many diverse industries” and cite many good examples, such as breakfast cereals, clothing, and tobacco industries.

The above considerations tell us that managers may be guided by sales or market share rather than pure profit maximization. Furthermore, the internal organizations of large firms usually involve a hierarchical structure, and perhaps the goals of the managers at each layer in the hierarchy are distinct from one another. If this is so, what is the optimal retail-organizational form for a firm? In this paper, we adopt a strategic approach and examine some implications of market size for the retail-organizational form of firms, where the forms include divisionalization, franchising, and mixing of divisions and franchises.

Recently, a group of papers (adopted a strategic approach to the retail-organizational forms,) focused on interactions among rivals in the market. In existing models, firms rely solely on divisionalization (Schwartz and Thompson, 1986; Corchon, 1991; Polasky, 1992; Baye et al., 1996a, b; Ziss, 1998; Corchon and Gonzalez-Maestre, 2000) or franchising (Wang and Wang, 2008; Saggi and Vettas, 2002) for expansion. However, the practice of expansion by a mix of divisions and franchises is visible in many industries, convenience stores and fast food chains being two examples. In the Cournot model, firms with lower marginal cost have higher output and profits. A franchise has to pay per-unit payments and therefore suffers a cost disadvantage (compared to a division). Accordingly, divisionalization aims to be aggressive toward rivals and benefits interbrand competition, while it also increases the competition between its own outlets (divisions and franchises). Since the marginal cost of a franchise is higher than that of a division, this allows for non-aggression
Divisionalization, Franchising, or Mixing: A Market Competition Perspective

159
toward rivals, whereas higher costs soften competition between outlets within the firm, thereby allowing it to harmonize intrabrand competition. In short, divisionalization is the interbrand advantage and intrabrand disadvantage option, whereas franchising is the reverse.

The objective of this paper is to examine how firms may rely on a mix of divisions and franchises for expansion when considering market competition. In our model, there are three layers in the hierarchy of each firm. At the top of the hierarchy is the top manager. Here, we consider three cases where top managers may seek to maximize sales, market share, and profit, respectively. The second layer of the hierarchy is the middle manager whose objective is profit maximization, because he will be rewarded according to his firm’s profit. The greater the profit, the higher the middle manager’s bonus. At the bottom of the hierarchy is the retail manager whose objective is maximizing the profit of his own outlet.

The company-owned outlets represent the spirit of a firm, and there is a strong incentive to retain more profitable sites as company-owned outlets (Martin, 1988). Accordingly, it is assumed that the top manager will choose the number of divisions to maximize his objective. After divisionalization is specified, the middle manager is delegated substantial authority over franchising. When franchising, middle managers choose royalty rates to harmonize intrabrand competition; that is, they wish their own retail managers will be less aggressive toward one another, thus relaxing fierce intrabrand competition for profit maximization. Finally, to exclude the case derived by Ziss (1998), Polasky (1992), and Corchon (1991), namely, that if demand is linear and divisionalization is costless, then a divisionalization equilibrium does not exist, we assume that each outlet involves a fixed cost and free entry is allowed to determine the number of franchises.

A mixing for expansion stems from the organizational conflict between the top manager and middle manager. If the objective of each top manager is sales maximization, and the market size is not sufficiently large, expansion by divisions alone will result in too fierce a competition, causing each retailer to make negative profit. Accordingly, top managers have no choice but to leave some room for franchises. When a firm’s divisionalization aims to maximize profits (market share) rather than sales revenue, firms do not franchise (divisionalize). This stems from the fact that the equilibrium outcome of profit delegation is more competitive than market share delegation, and market share delegation is more competitive than sales delegation, where firms rely on expansion by a mix of divisions and franchises.
Accordingly, firms do not franchise to avoid a cost disadvantage, provided that top managers are guided by simple profit maximization. When the objective of each top manager is market share maximization, all top managers expect their rivals will retaliate, and therefore, they reduce the temptation to be aggressive in the first place. Consequently, firms rely only on franchising for expansion to relax interbrand competition.

This paper is organized as follows. Section 2 describes the model we examine, Section 3 provides the results, and the final section states our conclusions.

2 The Model

Assume there are two firms, called A and B. Let \( n_t \) and \( m_t \) represent the number of firm \( T \)’s divisions and franchises, respectively, where \( T = A, B \). If \( Q_T \) represents the sum of output produced by these outlets, and if \( q_{ri}^o \) (\( q_{rj}^f \)) represents the output produced by the \( i^{th} \) (\( j^{th} \)) division (franchise) of firm \( T \), then \( Q_T = \sum_{i=1}^{n_t} q_{ri}^o + \sum_{j=1}^{m_t} q_{rj}^f \). Let \( Q_T \) denote the output of all outlets except those of firm \( T \), and let \( P_t(Q_T, Q_{-T}) \) denote the inverse demand function faced by all outlets of firm \( T \). We follow Ziss (1998), assuming that output is differentiated across firms but not across outlets of the same firm; that is, \( P_t(Q_t, Q_{-T}) = a - Q_T - sQ_{-T} \), where \( a > 0 \) represents the market size, and \( 0 \leq s \leq 1 \) measures the degree of substitution between the two products. Let \( f \) be the fixed cost of all outlets, and \( c \) be the marginal cost of all divisions, whereas a franchise of firm \( T \) will also be charged per-unit payments \( \theta_t \); in other words, the marginal cost of firm \( T \)’s franchise is \( c + \theta_t \). Let \( \pi_{ri}^o \) (\( \pi_{rj}^f \)) represent profits of the \( i^{th} \) (\( j^{th} \)) division (franchise) of firm \( T \):

\[
\pi_{ri}^o = (P_t - c)q_{ri}^o - f, \quad i = 1, 2, \ldots, n_t, \\
\pi_{rj}^f = (P_t - c - \theta_j)q_{rj}^f - f, \quad j = 1, 2, \ldots, m_t.
\]

Firm \( T \)’s sales are given by the sum of all sales create by its divisions and franchises, and firm \( T \)’s profits are the sum of the division profits and the per-unit payments paid from franchises. Firm \( T \)’s market share is measured by the fraction of industry sales. Let \( R_t \), \( \pi_t \), and \( S_t \) represent firm \( T \)’s sales, profits, and market share, respectively. Accordingly,
Divisionalization, Franchising, or Mixing: A Market Competition Perspective

\[ R_x = P_T \left( \sum_{i=1}^{n_x} q_{i}^p + \sum_{j=1}^{m_x} q_{j}^f \right), \quad \pi_x = \sum_{i=1}^{n_x} \pi_x^p + \theta_i \sum_{j=1}^{m_x} q_{j}^f \quad \text{and} \quad S_x = \frac{R_x}{R_x + R_y}. \]  

(3)

An expansion game consists of three stages. In the first stage, each top manager simultaneously and independently selects the number of divisions to maximize his objective. In the second stage, each middle manager likewise chooses per-unit payments to maximize his firm’s profits. In the last stage, each retail manager of both firms selects the output level to maximize the profits of his own outlet, and free entry is allowed.

### 2.1 Competition in Product Market

The model is solved using backward induction. In the third stage, each retail manager determines quantities to maximize the profits of his own outlet, and the number of franchises is determined by zero-profit condition, because free entry is allowed.\(^2\) In this stage we must determine each division’s choice of output, each franchise’s choice of output, and the two firms’ number of franchises. From Eq. (1), the decision of firm \(T\)’s division managers is as follows:

\[ \frac{d \pi_x^p}{d q_{j}^p} = P_T - c - q_{j}^p = 0, \quad T = A, B; \quad i = 1, 2, \ldots, n_x. \]

As all divisions of the same firm are identical, we denote the common output level of firm \(T\)’s divisions by \(q_{i}^c\), where \(q_{i}^c = q_{i1}^c = q_{i2}^c = \cdots = q_{im}^c\). Accordingly,

\[ P_T - c - q_{i}^c = 0, \quad T = A, B; \]

(4)

From Eq. (2), the choice of firm \(T\)’s franchise managers is as follows:

\[ \frac{d \pi_x^f}{d q_{j}^f} = P_T - c - \theta_j - q_{j}^f = 0, \quad T = A, B; \quad j = 1, 2, \ldots, m_x. \]

Similarly, we denote the common output level of firm \(T\)’s franchises by \(q_{j}^c\), where \(q_{j}^c = q_{j1}^c = q_{j2}^c = \cdots = q_{jm}^c\). Accordingly,

\(^2\)Since a franchise suffers a cost disadvantage compared to a division, the profit of a division is higher than that of a franchise. If the profit of a franchise is zero, then a division makes a nonnegative profit. But the reverse is not true. Accordingly, when free entry is allowed, the zero-profit condition determines the number of franchises. As to the number of divisions, it is determined in the first stage by the top manager.
The zero-profit condition is as follows:

\[(P_t - c - \theta_t) q_t^F - f = 0, \ T = A, B.\]  

(6)

Here we must determine \(q_A^o, q_B^o, q_A^r, q_B^r, m_A,\) and \(m_B.\) Eqs. (4), (5), and (6) include six equations; solving these equations yields:

\[q_t^o = \theta_t + \sqrt{f}, \ T = A, B,\]  

(7)

\[q_t^r = \sqrt{f}, \ T = A, B,\]  

(8)

\[m_r = \frac{(1-s)\alpha}{(1-s')\sqrt{f}} - \frac{\theta_r - s\theta_r}{(1-s')\sqrt{f}} - \frac{n_r\theta_r}{\sqrt{f}} - n_r, \ T = A, B; \ -T = A, B;\]

(9)

where \(\alpha = a - c - \sqrt{f}.\) Here \(m_r\) is firm \(T\)'s number of franchises. It is easily verified that the number of firm \(T\)'s franchises is negatively relative to its per-unit payments \((dm_r/d\theta_r < 0)\), and the number of firm \(T\)'s franchises is increasing with the rival's per-unit payments \((dm_r/d\theta_r > 0)\). Because the franchise fee raises the marginal cost of a franchise, this decreases its profitability. Accordingly, the higher the franchise fee, the lower the number of franchises. Secondly, when the number of firm \(T\)'s divisions increases, it is better to decrease the number of its own franchises \((dm_r/dn_r < 0)\) to avoid fierce intraindustrial competition. Notice that when \(s = 1,\) the above analysis is not appropriate, because the denominator equals zero in Eq. (9); we will demonstrate this case in Corollary 2.

### 2.2 Franchising by Per-unit Payments

We now go back to the franchising stage. In this stage, each middle manager specifies the royalty rate to maximize his firm’s profits. From Eq. (3) we can rewrite the profits as \(\pi_t = n_t(P_t - c)q_t^o - n_t f + m_t \theta_t q_t^r,\) and the first order condition is given by:

\[
\frac{d\pi_t}{d\theta_t} = \frac{\alpha}{1+s} - \frac{2\theta_t}{1-s} + \frac{s\theta_r}{1-s} + n_r \sqrt{f}.\]

(10)

Let \(d\pi_t/d\theta_t = 0\) and solve \(\theta_t\) and \(\theta_r\) to obtain:
\[
\theta_r = \frac{(1-s)\alpha}{2-s} + \frac{(1-s')(2n_r + s\alpha_s)}{4-s^2} \sqrt{f}, \quad T = A, B; \quad -T = A, B; \quad T \neq -T,
\]

where \( \theta_r \) is the franchise fee charged by firm \( T \). It is easily verified that the franchise fee is decreasing in its own, as well as its rival’s, number of divisions (\( d\theta_r/dn_r < 0, \quad d\theta_r/dn_{-r} < 0 \)). To avoid fierce intrabrand (interbrand) competition, when the number of its own (rival’s) divisions increases, it is better to raise the franchise fee to decrease the number of franchises.

### 2.3 Optimal Number of Divisions

We now proceed to the divisionalization stage. In this stage, each top manager chooses the number of divisions to maximize his objective. Here we consider three cases where top managers are guided by sales, market share, and profit, respectively.

#### 2.3.1 Sales Maximization

As Zabojnik (1998) points out, the salaries of top managers increase with firm sales, and it is therefore reasonable to assume that top managers are guided by sales maximization. From Eqs. (3), (7), and (8), firm \( T \) ’s sales can be rewritten as:

\[
R_r = n_r P_r q_r^* + m_r P_r q_r^* = P_r \left( (n_r + m_r) \sqrt{f} + n_r \theta_r \right).
\]

From Eq. (9), we know:

\[
n_r + m_r = \frac{(1-s)\alpha}{(1-s')} \frac{\theta_r - s \theta_{-r}}{\sqrt{f}} - \frac{n_r \theta_r}{(1-s') \sqrt{f}}.
\]

Hence, Eq. (12) can be rewritten as:

\[
R_r = P_r \left( \frac{1-s}{1-s'} \alpha - \frac{\theta_r - s \theta_{-r}}{1-s'} \right) = \frac{1}{1-s'} \left( (1-s)\alpha \theta_r - P_r \theta_r + s P_r \theta_{-r} \right).
\]

The first order condition is:

\[
\frac{dR_r}{dn_r} = \frac{1}{1-s'} \left( (1-s)\alpha \frac{dP_r}{dn_r} - \theta_r \frac{dP_r}{dn_r} - P_r \frac{d\theta_r}{dn_r} + s \theta_r \frac{dP_r}{dn_r} + s P_r \frac{d\theta_r}{dn_r} \right).
\]
Differentiating $P_r$, $θ_s$, and $θ_x$, with respect to $n_r$, and then applying symmetry by letting $n_r = n_s = n$ and $θ_s = θ_x = θ$ obtains:

$$\frac{dP_r}{dn_r} = \frac{dθ_s}{dn_r} = \frac{2(1-s^2)\sqrt{f}}{4-s^2} \quad \text{and} \quad \frac{dθ_x}{dn_r} = \frac{s(1-s^2)\sqrt{f}}{4-s^2}.$$ 

Accordingly,

$$\frac{dR_r}{dn_r} = \frac{1}{1-s} \left\{ \left( (1-s)α - (1-s)θ - P_r \right) \frac{dθ_s}{dn_r} + sP_r \frac{dθ_x}{dn_r} \right\}. \tag{14}$$

Substituting $dθ_s/dn_r$ and $dθ_x/dn_r$ into Eq. (14) yields:

$$\frac{dR_r}{dn_r} = \frac{\sqrt{f}}{4-s^2} \left[ 2(1-s)α - (4-2s-s^2)(c + \sqrt{f} + θ) \right]. \tag{15}$$

From Eq. (11), we know $θ = [(1-s)α + (1-s^2)n\sqrt{f}]/(2-s)$, and substituting into Eq. (15), we have:

$$\frac{dR_r}{dn_r} = \frac{\sqrt{f}\beta}{(4-s^2)(2-s)}, \tag{16}$$

where $β = s^2(1-s)α - (4-2s-s^2)(c + \sqrt{f}) - (1-s^2)(4-2s-s^2)n\sqrt{f}.$

Let $n^*$, $θ^*$, $m^*$, $q^*$, $q^{**}$, and $P^*$ denote the equilibrium number of divisions, the franchise fee, the number of franchises, the output of each division, the output of each franchise and market price, respectively. By letting Eq. (16) equal zero, we have $n^*$. Substituting $n^*$ into Eq. (11) gives us $θ^*$. By substituting $n^*$ and $θ^*$ into Eq. (9), we obtain $m^*$. Eqs. (7) and (8) give us $q^{**}$ and $q^{***}$, respectively. Finally, from the inverse demand function, we have $P^*$. \[3\]

### 2.3.2 Market Share Maximization

In this subsection, we postulate that the objective of the top manager is to maximize market share, as in Ritz (2008), Wang and Wang (2008) and Jansen et al. (2007). Let $S_r$ be firm $T$’s market share. In the first stage, each top manager chooses the number of divisions to maximize his firm’s market share, which is measured as

\[3\] Since the Nash equilibrium of each subgame is unique, the subgame-perfect Nash equilibrium is unique. In Appendix, it is verified that the second-order conditions are satisfied.
Divisionalization, Franchising, or Mixing: A Market Competition Perspective

follows:

\[ S_r = \frac{R_r}{R_r + R_{-r}}. \]

The first order condition is:

\[
\frac{dS_r}{dn_r} = \frac{dR_r}{dn_r} \frac{(R_r + R_{-r}) - R_r \left( \frac{dR_r}{dn_r} + \frac{dR_{-r}}{dn_r} \right)}{(R_r + R_{-r})^2} = \frac{dR_r}{dn_r} \frac{R_r - R_{-r}}{(R_r + R_{-r})^2}.
\]

Owing to symmetry, we have \( R_r = R_{-r} = R \); hence,

\[
\frac{dS_r}{dn_r} = R \left( \frac{dR_r}{dn_r} - \frac{dR_{-r}}{dn_r} \right) (R_r + R_{-r})^{-2}.
\]

From Eq. (14), we know:

\[
\left( \frac{(1-s)\alpha}{1-s^2} \right) \left( \frac{(1-s)\theta - P_r}{1-s^2} \right) \frac{d\theta}{dn_r} + sP_r \frac{d\theta}{dn_r} \right].
\]

Accordingly,

\[
\frac{dR_{-r}}{dn_r} = \frac{1}{1-s^2} \left[ (1-s)\alpha -(1-s)\theta - P_r \right] \frac{d\theta}{dn_r} + sP_r \frac{d\theta}{dn_r} \right].
\]

Owing to symmetry, we have \( P_r = P_{-r} = c + \sqrt{f + \theta} \); hence,

\[
\frac{dR_r}{dn_r} - \frac{dR_{-r}}{dn_r} = \frac{2-s}{4-s^2} (-s(1-s)\alpha - 2(2-s)(c + \sqrt{f}) - 2(1-s^2)n\sqrt{f}) < 0.
\]

That is,

\[
\frac{dS_r}{dn_r} < 0.
\]

If top managers are guided by market share, then setting up more divisions would increase the firm’s market share, and rivals would retaliate \( dR_{-r}/dn_r > 0 \). All top managers expect this reaction, so no top manager has an incentive to set up more divisions only to earn lower profits after retaliation. Accordingly, top managers do not
set up divisions, and firms only rely on franchising for expansion.4

2.3.3 Profit Maximization

Now we consider when top managers are guided by pure profit maximization. From Eq. (3), we can rewrite $\pi_r$ as follows:

$$\pi_r = n_r(P_r - c)q_r^n - n_r f + m_r \theta_r q_r^n = n_r(P_r - c)(\theta_r + \sqrt{f}) - n_r f + m_r \theta_r \sqrt{f}.$$ 

Substituting $P_r = c + \theta_r + \sqrt{f}$ into $\pi_r$, we have:

$$\pi_r = n_r(\theta_r + \sqrt{f} + \theta_r \sqrt{f}) - n_r f + m_r \theta_r \sqrt{f}$$

$$= n_r(\theta_r^2 + 2\theta_r \sqrt{f} + f) - n_r f + m_r \theta_r \sqrt{f}$$

$$= \theta_r(n_r + m_r)\sqrt{f} + n_r \theta_r \sqrt{f}.$$ 

From Eq. (9), we know:

$$(m_r + n_r)\sqrt{f} + n_r \theta_r = \frac{(1-s)\alpha}{1-s^2} = -\frac{\theta_r - s \theta_x}{1-s^2}.$$ 

Accordingly,

$$\pi_r = \theta_r \left(\frac{(1-s)\alpha}{1-s^2} - \frac{\theta_r - s \theta_x}{1-s^2}\right) + n_r \theta_r \sqrt{f}.$$ 

Differentiate $\pi_r$ with respect to $n_r$ yields:

$$\frac{d\pi_r}{dn_r} = \frac{d\theta_r}{dn_r} \left(\frac{(1-s)\alpha}{1-s^2} - \frac{\theta_r - s \theta_x}{1-s^2}\right) - \frac{\theta_r}{1-s^2} \left(\frac{d\theta_r}{dn_r} - \frac{s d\theta_x}{dn_r}\right) + \theta_r \sqrt{f} + n_r \sqrt{f} \frac{d\theta_x}{dn_r}.$$ 

Differentiating $\theta_r$ and $\theta_x$ with respect to $n_r$, and then applying symmetry by letting $n_r = n_x = n$ and $\theta_r = \theta_x = 0$ obtains:

$$\frac{d\theta_r}{dn_r} = \frac{2(1-s^2)\sqrt{f}}{4-s^2} \quad \text{and} \quad \frac{d\theta_x}{dn_r} = \frac{s(1-s^2)\sqrt{f}}{4-s^2}.$$ 

Substituting the above equations into $d\pi_r/dn_r$ gives us:

---

4Even if we measure the market share as in Ritz (2008), Jansen et al. (2007), that is, firm $T$'s market share is measured by the fraction of industry output levels, the result $dS_t/dn_r < 0$ still holds.
When the objective of the top manager is profit maximization, firms rely only on divisionalization for expansion. The reason is similar to why firms do not divisionalize when top managers are guided by market share. It might seem that market share and sales delegation would be more competitive than profit maximization. However, market share and sales delegation reduce the degree of interdependence, and hence, the temptation to be aggressive in the first place (Ritz, 2008). Accordingly, the outcome of market share and sales delegation is less competitive than profit maximization. That is, profit maximization is the most competitive environment. Because a franchise has to pay per-unit payments, it will suffer a cost disadvantage, therefore, top managers will leave no opportunity for franchises, and firms will not franchise.

3 The Results

In the previous section, we analyzed the expansion game. When top managers are guided by sales maximization, a firm might rely on a mix of divisions and franchises for expansion. When the two products are substitutes and the market size is moderate, we can show that mixing is the best choice for firms. That is,

**Proposition 1.** When \((4 - 2s - s^2)(c + \sqrt{f}) < s^2(1-s)a < \Omega\), where \((4 - 2s - s^2) \times (c + (2 - s^2/2)\sqrt{f}) < \Omega < (4 - 2s - s^2)(c + 2\sqrt{f})\), we have:

\[
\begin{align*}
n^* &= \frac{s^2(1-s)a - (4 - 2s - s^2)(c + \sqrt{f})}{(1-s^2)(4 - 2s - s^2)\sqrt{f}}, \\
\theta^* &= \frac{2(1-s)a}{4 - 2s - s^2} - (c + \sqrt{f}), \\
m^* &= \frac{\theta^*[1 - (1-s^2)n^*]}{(1-s^2)\sqrt{f}} - 2n^*, \text{ and} \\
P^* &= \frac{2(1-s)a}{4 - 2s - s^2}.
\end{align*}
\]

**Proof.** It is easy to verify \(n^*, \theta^*, m^*, \) and \(P^*\). The condition \((4 - 2s - s^2) \times (c + \sqrt{f}) < s^2(1-s)a \) implies \(n^* > 0\). To get \(m^* > 0\), we must have \(\theta^* >
\[2(1-s')/\sqrt{1 - (1-s')n'} \]. Now,
\[
\theta' = \frac{2(1-s)a}{4-2s-s^2} - (c + \sqrt{f}) = \frac{2(1-s)(\sqrt{f} + (2-s')c + \sqrt{f})}{s^2} > \frac{2(1-s)\sqrt{f}n' + (2-s')\sqrt{f}}{s^2}.
\]

Let
\[
\Delta = \frac{2(1-s)\sqrt{f}n' + (2-s')\sqrt{f}}{s^2} - \frac{2(1-s')\sqrt{f}n'}{1-(1-s')n'} = \frac{\sqrt{f}(2(1-s')n' - (2-s'))[(1-s')n' + 1]}{s^2 [1-(1-s')n']^2}.
\]

Now, \( \Delta = 0 \) implies \( \theta' > 2(1-s')/\sqrt{1 - (1-s')n'} \), and consequently \( m' > 0 \). Secondly, \( a = (4-2s-s^2)(c + (2-s'/2)\sqrt{f})/s^2(1-s) \) implies \( n' = (2-s')/[2(1-s')] \), and \( \Delta = 0 \). In short, \( s'(1-s)a = (4-2s-s^2)\times (c + (2-s'/2)\sqrt{f}) \) implies \( m' > 0 \).

It is easily shown that \( dm'/da < 0 \); hence, there exists \( (4-2s-s^2)\times (c + (2-s'/2)\sqrt{f}) \leq \Omega \) such that \( (4-2s-s^2)(c + (2-s'/2)\sqrt{f}) < s'(1-s)a < \Omega \) implies \( m' = 0 \). Finally, the condition \( s'(1-s)a < (4-2s-s^2)(c + 2\sqrt{f}) \) implies \( 1-(1-s')n' < 0 \), and this results in \( m' < 0 \). Consequently, we must have \( \Omega < (4-2s-s^2)(c + 2\sqrt{f}) \) to ensure \( m' > 0 \).

Proposition 1 indicates that if the market size is neither too small \( (4-2s-s^2)(c + \sqrt{f})/s^2(1-s) < a \) nor too large \( a < \Omega/s^2(1-s) \), then firms prefer a mix of divisions and franchises.\(^5\) The intuition is that if firms only consider the interbrand competition and just set up divisions, this is too aggressive for the moderate market to bear, and expansion only by franchising is too passive to enlarge sales. That is, on the one hand, top managers might worry about too many divisions of all firms combined, where each retailer makes a negative profit. On the other hand, top managers might worry that they have too few outlets, so that to avoid fierce intrabrand

\(^5\) Specifically, four factors impact the choice of the organizational form: market size, marginal cost, fixed cost, and substitutability. The fixed cost is relatively small for a large market, so the upper (lower) bound of market size is similar to the condition that fixed cost must be large (small) enough. Similarly, the marginal cost is relatively small for a large market. As to the substitutability, we can rewrite the condition in Proposition 1 as \( (c + \sqrt{f})/a < s'(1-s)(4-2s-s') < (c + (2-s'/2)\sqrt{f})/a \), accordingly, firms may rely on a mix for expansion provided that \( 0 < s < 1 \).
Divisionalization, Franchising, or Mixing: A Market Competition Perspective

169

...competition they have no choice but to franchise. Consequently, top managers will leave some room for franchises, and a mix of divisions and franchises might be the optimal retail-organizational form.

Proposition 2. When \( s^2(1-s) a \leq (4-2s-s^3)(c + \sqrt{f}) \), then the equilibrium number of divisions \( n' = 0 \), and

\[
\theta' = \frac{(1-s)(a - c - \sqrt{f})}{2 - s},
\]

\[
m' = \frac{a - c - \sqrt{f}}{(1+s)(2-s)\sqrt{f}}, \quad \text{and}
\]

\[
p' = \frac{(1-s)a + c + \sqrt{f}}{2 - s}.
\]

Proof. Eq. (16) implies that the Nash equilibrium number of divisions is non-positive for \( s^2(1-s) a \leq (4-2s-s^3)(c + \sqrt{f}) \). Let \( n_0 \) be the solution of \( dR_t/dn_t = 0 \), then \( n_0 < 0 \), and \( R_t \) is decreasing in \( n_t \) for \( n_t < n_0 \), thus, \( n' = 0 \). And \( \theta' \), \( m' \), and \( p' \) are as required.

Corollary 1. If \( s = 0 \), then \( n' = 0 \), \( \theta' = (a - c - \sqrt{f})/2 \), \( m' = (a - c - \sqrt{f})/2\sqrt{f} \), and \( p' = (a + c + \sqrt{f})/2 \).

When the market is small, there are no incentives for top managers to fight for sales, and the marginal cost of a franchise is higher than that of a division, hence setting up franchises rather than divisions can alleviate intrabrand competition, therefore firms rely on expansion only by franchises. If the products are independent in terms of demand, that is \( s = 0 \), there is no interbrand competition. Similarly, firms rely on franchises alone to relax intraband competition.

Proposition 3. When \( \Omega \leq s^2(1-s) a \), the optimal number of franchises \( m' = 0 \), and

\[
n' = (a - c - \sqrt{f})/[(1+s)\sqrt{f}], \quad q' = \sqrt{f}, \quad \text{and} \quad p' = c + \sqrt{f}.
\]

Proof. When \( \Omega \leq s^2(1-s) a \), if \( n' \) and \( \theta' \) are chosen as in proposition 1, then \( m' \leq 0 \). Consequently, firms only set up divisions to flood the market until each firm’s profits are zero. When \( m' = 0 \), the game reduces to two-stage game. In stage one, each top manager chooses the number of divisions to maximize sales revenue subject to zero-profit constraint. In stage two, each division manager maximizes the division’s profits.
Now firm $T$’s profits are reduced to $\pi_T = \sum_{i=1}^{n} \pi_{Ti}$, and each division’s profits are the same as Eq. (1). Differentiating Eq. (1) with respect to $q_{Ti}$, and then applying symmetry by letting $q_{Ti}^* = q_{n_i}^*$ for every $i$ yields:

$$q_{Ti}^* = \frac{(a-c)(n_{Ti} + 1 - sn_{Ti})}{(n_{Ti} + 1)(n_{Ti} + 1 - s'n_{Ti})}.$$ (17)

We now go back to the first stage. In this case, the maximization problem of firm $T$’s top manager is:

$$\max_{n_T} R_T = n_T q_T^{\partial}$$

$s.t. \quad n_T [(P_T - c)q_T - f] = 0$.

Substituting $P_T q_T^{\partial} = cq_T^{\partial} + f$ into the objective function, then $R_T$ can be rewritten as $n_T (cq_T^{\partial} + f)$. Consequently, the first order condition is:

$$\frac{dR_T}{dn_T} = cq_T^{\partial} + f + n_T c \frac{dq_T^{\partial}}{dn_T}.$$ 

Applying symmetry by letting $n = n_s = n_s$, we have:

$$\frac{dR_T}{dn_T} = c(a-c)(n+1) + f > 0.$$ 

Hence, firms increase the number of divisions until each division’s profits are zero. Owing to $n_s = n_s$, each division of both firms produces the same output level. Let $q = q_s^* = q_s^{\partial}$ and then substitute into zero-profit condition to obtain:

$$[a-c-(1+s)nq]q - f = 0.$$ (18)

Substitute Eq. (17) into Eq. (18) and then solve $n$ to yield:

$$n^* = \frac{a-c-\sqrt{f}}{(1+s)\sqrt{f}}.$$ 

Substitute $n^*$ into Eq. (17) to obtain $q^{\partial s} = \sqrt{f}$, and $P^* = c + q^{\partial s} = c + \sqrt{f}$. 

**Corollary 2.** If $s = 1$, then $m^* = 0$, $n^* = (a - c - \sqrt{f})/2\sqrt{f}$, $q^{\partial s} = \sqrt{f}$ and $P^* = c + \sqrt{f}$.

**Proof.** When $s = 1$, the two products are perfect substitutes. Under free entry each
franchise of firm $T$ will make a negative profit provided that $\theta_r > \theta_s$. This means $m_r = 0$ as $\theta_r > \theta_s$. As a consequence, in the franchising stage, the two middle managers are likely to play a Bertrand game. In equilibrium both middle managers charge the same royalty rates and $\theta_r = \theta_s = 0$, therefore, firms do not franchise. Now our game is the same as the reduced game in Proposition 3, and substituting $s = 1$ into Proposition 3 we obtain the result.

There exist very strong incentives for top managers to fight for sales when the market is sufficiently large. It is better for a top manager just setting up divisions to avoid being at a disadvantage towards rivals. For the same reason, if the two products are perfect substitutes, it is better for firms to divisionalize rather than franchise. Here we give numerical examples to show the results stated by Propositions 1, 2, and 3. That is, only when the market is medium-sized can firms rely on a mix of divisions and franchises for expansion. If the market size is large (small) enough, then firms rely only on divisionalization (franchising) for expansion. Let $s = 19/20$, $c = 0$ and $f = 100$, we have

Table 1: Numerical Examples for Propositions 1, 2 and 3

<table>
<thead>
<tr>
<th>$a$</th>
<th>200</th>
<th>250</th>
<th>300</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
<th>550</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{n}$</td>
<td>0</td>
<td>0</td>
<td>1.34</td>
<td>3.27</td>
<td>5.20</td>
<td>22.56</td>
<td>25.13</td>
<td>27.69</td>
<td>30.26</td>
</tr>
<tr>
<td>$\bar{m}$</td>
<td>9.28</td>
<td>11.72</td>
<td>10.74</td>
<td>6.89</td>
<td>1.43</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\theta'$</td>
<td>9.05</td>
<td>11.43</td>
<td>15.05</td>
<td>19.23</td>
<td>23.40</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{P}$</td>
<td>19.05</td>
<td>21.43</td>
<td>25.05</td>
<td>29.23</td>
<td>33.40</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Finally, we will examine the competitiveness of the three different incentives.

**Proposition 4.** The equilibrium outcome of profit delegation is more competitive than market share delegation, and market share delegation is more competitive than sales delegation where firms rely on expansion by a mix of divisions and franchises.

**Proof.** Let $P^\alpha$, $P^\tau$ and $P^\theta$ be the equilibrium prices of profit delegation, market share delegation and sales delegation, respectively. We know when top managers are guided by profit maximization, firms will not franchise, that is, $P^\alpha = c + \sqrt{f}$ by Proposition 3. When the objective of each top manager is market share maximization, then firms will not divisionalize, that is $P^\tau = (1-s)a + c + \sqrt{f}/2 - s$ by Proposition 2. When top managers are guided by sales maximization, and market size is moderate, i.e. $(4-2s-s^2)(c+\sqrt{f})<s^2(1-s)a<\Omega$, then $P^\theta = 2(1-s)a/(4-2s-s^2)$ by
Proposition 1. It is easily verified that $P^\alpha < P^\nu < P^\sigma$:

$$
P^\nu - P^\alpha = \frac{(1-s)a + c + \sqrt{f}}{2-s} - (c + \sqrt{f}) = \frac{(1-s)(a - (c + \sqrt{f}))}{2-s} > 0,
$$
and

$$
P^\nu - P^\sigma = \frac{2(1-s)a}{4-2s-s^2} - \frac{(1-s)a + c + \sqrt{f}}{2-s}
= \frac{s^2(1-s)a - (4-2s-s^2)(c + \sqrt{f})}{4-2s-s^2} > 0.
$$

Ritz (2008) and Jansen et al. (2007) pointed out that the equilibrium outcome of market share incentive is less competitive than sales incentive. It stems from the fact that it would be more aggressive when top managers are guided by market share. However, all top managers expect their rivals would retaliate and hence they reduce the temptation to be aggressive in the first place. For the same reason, the equilibrium outcome of sales incentive is less competitive than profit incentive.

Our result is distinct from Ritz (2008) and Jansen et al. (2007) when comparing the equilibrium outcome of sales delegation with market share delegation. The reason is that the number of competing units is exogenous in Ritz (2008) and Jansen et al. (2007). That is, in their model the number of firms is the same in different delegations. But the number of franchises (divisions) is endogenous in our model and zero profit condition determines the number of franchises. If top managers are guided by market share, they only set up franchises. Accordingly, all competing units make zero profit in equilibrium. However, if top managers are delegated by sales revenue and rely on a mix of divisions and franchises for expansion, then each division makes a positive profit in equilibrium for its cost advantage compared to a franchise. Hence, sales incentive is less competitive than market share incentive.

4 Conclusions

We adopt the strategic approach to explain why a mix of divisions and franchises is desirable for a firm. It is better for firms to rely on this kind of mix when there exists hierarchical conflict, the products are substitutes, and the market size is moderate. This concerns a tradeoff between interbrand and intrabrand competition, and a mix of divisions and franchises can exploit both interbrand and intrabrand advantages.
The sales maximization for divisionalization plays a key role in the choice of retail-organizational form. If a firm’s divisionalization aims to maximize profits (market share), then firms do not franchise (divisionalize). This is because the equilibrium outcome of profit delegation is more competitive than that of market share delegation, which itself is more competitive than that of sales delegation where firms rely on expansion by a mix of divisions and franchises. Consequently, firms do not franchise to avoid a cost disadvantage, provided that top managers are guided by simple profit maximization. When the objective of each top manager is market share maximization, all top managers expect their rivals will retaliate, and hence, they reduce the temptation to be aggressive in the first place. Accordingly, firms do not divisionalize to soften interbrand competition.

Appendix

It’s easily to show \( d^2 \pi^o / d(q^o)^2 = d^2 \pi^t / d(q^t)^2 = -2 \) and \( d^2 \pi^t / d\theta^t = -2(1-s^2) \).

Eq. (11) gives us \( d\theta^t / dn_r = 2(1-s^2) \sqrt{f / 4 - s^2} \), \( d\theta^t \theta ^t / dn_r = s(1-s^2) \sqrt{f / 4 - s^2} \) and \( d^2 \theta^t / dn_r^2 = d^2 \theta^t / dn_r = 0 \). Owing to:

\[
\frac{dR_r}{dn_r} = \frac{1}{1-s^2} \left[ (1-s)\frac{dP_r}{dn_r} - \theta^t \frac{dP_r}{dn_r} - P_r \frac{d\theta^t}{dn_r} + s \theta^t \frac{dP_r}{dn_r} + sP_r \frac{d\theta^t}{dn_r} \right],
\]

therefore,

\[
\frac{d^2 R_r}{dn_r^2} = \frac{1}{1-s^2} \left[ -\left( \frac{d\theta^t}{dn_r} \right)^2 \right] - \left( \frac{d\theta^t}{dn_r} \right)^2 s \left( \frac{d\theta^t}{dn_r} \right)^2 + s \left( \frac{d\theta^t}{dn_r} \right)^2 (1-s^2) \left( \frac{d\theta^t}{dn_r} \right)^2 + s \left( \frac{d\theta^t}{dn_r} \right)^2 (1-s^2) \left( \frac{d\theta^t}{dn_r} \right)^2 \right]
\]

\[
= -\frac{2}{1-s^2} \left( \frac{d\theta^t}{dn_r} \right)^2 \left( \frac{d\theta^t}{dn_r} - s \frac{d\theta^t}{dn_r} \right)
\]

\[
= -\frac{2}{1-s^2} \left( \frac{d\theta^t}{dn_r} \right)^2 \left[ (1-s^2) \sqrt{f / 4 - s^2} - \frac{(1-s^2)s^2 \sqrt{f / 4 - s^2}}{4-s^2} \right] = -\frac{2}{1-s^2} \left( \frac{d\theta^t}{dn_r} \right)^2 < 0.
\]

Reference


